AMERICAN MATHEMATICS COMPETITIONS

1st Annual American Mathematics Contest →10

AMC → 10

TUESDAY, FEBRUARY 15, 2000

Sponsored by
Mathematical Association of America
Society of Actuaries   Mu Alpha Theta
National Council of Teachers of Mathematics
Casualty Actuarial Society   American Statistical Association
American Mathematical Association of Two-Year Colleges
American Mathematical Society
American Society of Pension Actuaries
Consortium for Mathematics and its Applications
Pi Mu Epsilon   National Association of Mathematicians
School Science and Mathematics Association
Clay Mathematics Institute
University of Nebraska-Lincoln

1. DO NOT OPEN THIS BOOKLET UNTIL TOLD TO DO SO BY YOUR PROCTOR.
2. This is a twenty-five question, multiple choice test. Each question is followed by answers marked A,B,C,D and E. Only one of these is correct.
3. The answers to the problems are to be marked on the AMC → 10 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. SCORING: You will receive 6 points for each correct answer, 2 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, ruler, compass, protractor, erasers and calculators that are accepted for use on the SAT. No problems on the test will *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form. When your proctor gives the signal, begin working the problems. You will have **75 MINUTES** working time to complete the test.
8. When you finish the exam, *sign your name* in the space provided on the Answer Form.

Students who score in the top 1% on this AMC →10 will be invited to take the 18th annual American Invitational Mathematics Examination (AIME) on Tuesday, March 28, 2000. More details about the AIME and other information are on the back page of this test booklet.

The Committee on the American Mathematics Competitions (CAMC) reserves the right to re-examine students before deciding whether to grant official status to their scores. The CAMC also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

1. In the year 2001, the United States will host the International Mathematical Olympiad. Let I, M, and O be distinct positive integers such that the product $I \cdot M \cdot O = 2001$. What is the largest possible value of the sum $I + M + O$?

   (A) 23 **(B)** 55 **(C)** 99 **(D)** 111 **(E)** 671

2. $2000(2000^{2000}) =$

   (A) 2000^{2001} **(B)** 4000^{2000} **(C)** 2000^{4000}

   (D) $4{,}000{,}000^{2000}$ **(E)** $2000^{4{,}000{,}000}$

3. Each day, Jenny ate 20% of the jellybeans that were in her jar at the beginning of that day. At the end of second day, 32 remained. How many jellybeans were in the jar originally?

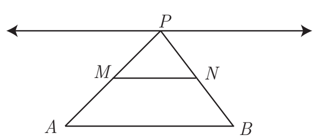
   (A) 40 **(B)** 50 **(C)** 55 **(D)** 60 **(E)** 75

4. Chandra pays an on-line service provider a fixed monthly fee plus an hourly charge for connect time. Her December bill was \$12.48, but in January her bill was \$17.54 because she used twice as much connect time as in December. What is the fixed monthly fee?

   (A) \$2.53 **(B)** \$5.06 **(C)** \$6.24 **(D)** \$7.42 **(E)** \$8.77

5. Points M and N are the midpoints of sides PA and PB of $\triangle PAB$. As P moves along a line that is parallel to side AB, how many of the four quantities listed below change?

   (a) the length of the segment MN
   (b) the perimeter of $\triangle PAB$
   (c) the area of $\triangle PAB$
   (d) the area of trapezoid $ABNM$

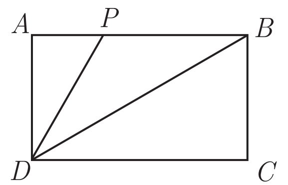


   (A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

6. The Fibonacci sequence $1, 1, 2, 3, 5, 8, 13, 21, \ldots$ starts with two 1s, and each term afterwards is the sum of its two predecessors. Which one of the ten digits is the last to appear in the units position of a number in the Fibonacci sequence?

   (A) 0 **(B)** 4 **(C)** 6 **(D)** 7 **(E)** 9

7. In rectangle $ABCD$, $AD = 1$, P is on $\overline{AB}$, and $\overline{DB}$ and $\overline{DP}$ trisect $\angle ADC$. What is the perimeter of $\triangle BDP$?



(A) $3 + \frac{\sqrt{3}}{3}$ **(B)** $2 + \frac{4\sqrt{3}}{3}$ **(C)** $2 + 2\sqrt{2}$
(D) $\frac{3 + 3\sqrt{5}}{2}$ **(E)** $2 + \frac{5\sqrt{3}}{3}$

8. AT Olympic High School, 2/5 of the freshmen and 4/5 of the sophomores took the **AMC→10**. Given that the number of freshmen and sophomore contestants was the same, which of the following must be true?

(A) There are five times as many sophomores as freshmen.

(B) There are twice as many sophomores as freshmen.

(C) There are as many freshmen as sophomores.

(D) There are twice as many freshmen as sophomores.

(E) There are five times as many freshmen as sophomores.

9. If $|x - 2| = p$, where $x < 2$, then $x - p =$

(A) -2 **(B)** 2 **(C)** $2 - 2p$ **(D)** $2p - 2$ **(E)** $|2p - 2|$

10. The sides of a triangle with positive area have lengths 4,6, and x. The sides of a second triangle with positive area have lengths 4, 6, and y. What is the smallest positive number that is **not** a possible value of $|x - y|$?

(A) 2 **(B)** 4 **(C)** 6 **(D)** 8 **(E)** 10

11. Two different prime numbers between 4 and 18 are chosen. When their sum is subtracted from their product, which of the following number could be obtained?

(A) 21 **(B)** 60 **(C)** 119 **(D)** 180 **(E)** 231

12. Figure 0,1,2, and 3 consist of 1,5,13, and 25 nonoverlapping unit squares, respectively. If the pattern were continued, how many nonoverlapping unit squares would there be in figure 100 ?

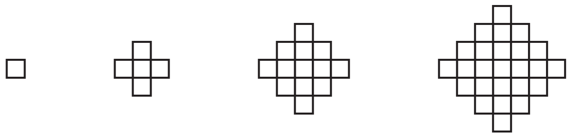
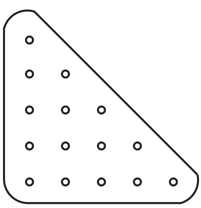


figure 0 figure 1 figure 2 figure 3

(A) 10401 **(B)** 19801 **(C)** 20201 **(D)** 39801 **(E)** 40801

13. There are 5 yellow pegs, 4 red pegs, 3 green pegs, 2 blue pegs, and 1 orange peg to be placed on a triangular peg board. In how many ways can the pegs be placed so that no (horizontal) row or (vertical) column contains two pegs of the same color?



(A) 0 **(B)** 1 **(C)** $5! \cdot 4! \cdot 3! \cdot 2! \cdot 1!$

(D) $15!/(5! \cdot 4! \cdot 3! \cdot 2! \cdot 1!)$ **(E)** $15!$

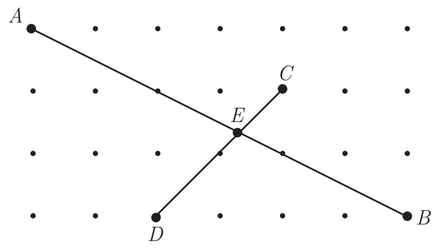
14. Mrs.Walter gave an exam in a mathematics class of five students. She entered the scores in random order into a spreadsheet, which recalculated the class average after each score was entered. Mrs.Walter noticed that after each score was entered, the average was always an integer. The scores (listed in ascending order) were 71, 76, 80, 82, and 91. What was the last scores Mrs.Walter entered?

(A) 71 **(B)** 76 **(C)** 80 **(D)** 82 **(E)** 91

15. Two non-zero real numbers, a and b, satisfy $ab = a - b$. Find a possible value of $\frac{a}{b} + \frac{b}{a} - ab$.

(A) -2 **(B)** $-\frac{1}{2}$ **(C)** $\frac{1}{3}$ **(D)** $\frac{1}{2}$ **(E)** 2

16. The diagram shows 28 lattice points, each one unit from its nearest neighbors. Segment AB meets segment CD at E. Find the length of segment AE.



(A) $4\sqrt{5}/3$ **(B)** $5\sqrt{5}/3$ **(C)** $12\sqrt{5}/7$ **(D)** $2\sqrt{5}$ **(E)** $5\sqrt{65}/9$

17. Boris has an incredible coin changing machine. When he puts in a quarter, it returns five nickels; when he puts in a nickel, it returns five pennies; and when he puts in a penny, it returns five quarters. Boris starts with just one penny. Which of the following amounts could Boris have after using the machine repeatedly?

(A) \$3.63 **(B)** \$5.13 **(C)** \$6.30 **(D)** \$7.45 **(E)** \$9.07

18. Charlyn walks completely around the boundary of a square whose sides are each 5 km long. From any point on her path she can see exactly 1 km horizontally in all directions. What is the area of the region consisting of all points Charlyn can see during her walk, expressed in square kilometers and rounded to the nearest whole number?

(A) 24 **(B)** 27 **(C)** 39 **(D)** 40 **(E)** 42

19. Through a point on the hypotenuse of a right triangle, lines are drawn parallel to the legs of the triangle so that the triangle is divided into a square and two smaller right triangles. The area of one of the two small right triangle is m times the area of the square. The ratio of the area of the other small right triangle to the area of the square is

(A) $\dfrac{1}{2m+1}$ **(B)** m **(C)** $1-m$ **(D)** $\dfrac{1}{4m}$ **(E)** $\dfrac{1}{8m^2}$

20. Let A, M, and C be nonnegative integers such that $A+M+C=10$. What is the maximum value of $A\cdot M\cdot C+A\cdot M+M\cdot C+C\cdot A$?

(A) 49 **(B)** 59 **(C)** 69 **(D)** 79 **(E)** 89

21. If all alligators are ferocious creatures and some creepy crawlers are alligators, which statement(s) **must** be true?

    I. All alligators are creepy crawlers.

    II. Some ferocious creatures are creepy crawlers.

    III. Some alligators are not creepy crawlers.

    (A) I only **(B)** II only **(C)** III only

    (D) II and III only **(E)** None must be true

22. One morning each member of Angela's family drank an 8-ounce mixture of coffee with milk. The amounts of coffee and milk varied from cup to cup, but were never zero. Angela drank a quarter of the total amount of milk and a sixth of the total amount of coffee. How many people are in the family?

    (A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

23. When the mean, median, and mode of the list

$$10, 2, 5, 2, 4, 2, x$$

are arranged in increasing order, they form a non-constant arithmetic progression. What is the sum of all possible real value of x?

    (A) 3 **(B)** 6 **(C)** 9 **(D)** 17 **(E)** 20

24. Let f be a function for which $f(x/3) = x^2 + x + 1$. Find the sum of all values of z for which $f(3z) = 7$.

    (A) $-1/3$ **(B)** $-1/9$ **(C)** 0 **(D)** $5/9$ **(E)** $5/3$

25. In year N, the 300^{th} day of the year is a Tuesday. In year $N+1$, the 200^{th} day is also a Tuesday. On what day of the week did the 100^{th} day of year $N-1$ occur?

    (A) Thursday **(B)** Friday **(C)** Saturday **(D)** Sunday **(E)** Monday

WRITE TO US!

Correspondence about the problems and solutions for this AMC → **10** should be addressed to:

Professor Harold B. Reiter, AMC →10, Chair
Dept. of Mathematics, University of North Carolina, Charlotte, NC 28223 USA
eMail: hbreiter@email.uncc.edu; Web: http://www.math.uncc.edu/hbreiter
Phone: 704-547-4561; Home: 704-364-5699; Fax: 704-510-6415

Orders for any publications listed below should be addressed to:

Titu Andreescu, Director
American Mathematics Competitions
University of Nebraska, P.O. Box 81606
Lincoln, NE 68501-1606
Phone: 402-472-2257; Fax: 402-472-6087; eMail: titu@amc.unl.edu; www.unl.edu/amc

2000 AIME

The AIME will be held on Tuesday, March 28, 2000. It is a 15-question, 3-hour, integer-answer exam. You will be invited to participate only if you score in the top 1% of this AMC →10 or receive a score of 100 or above on the AMC →12. Top-scoring students on the AMC →10/12/AIME will be selected to take the USA Mathematical Olympiad (USAMO) on Tuesday, May 2, 2000. The best way to prepare for the AIME and USAMO is to study previous years of these exams. Copies may be ordered as indicated below.

PUBLICATIONS

MINIMUM ORDER: $10 (before shipping/handling fee), *PAYMENT IN US FUNDS ONLY made payable to the* American Mathematics Competitions or VISA/MASTERCARD accepted. Include card number, expiration date, cardholder name and address. U.S.A. and Canadian orders must be prepaid and will be shipped Priority Mail, UPS or Air Mail.
INTERNATIONAL ORDERS: Do **NOT** prepay. An invoice will be sent to you.

Examinations: Each price is for one copy of an exam and its solutions for one year. Specify the years you want and how many copies of each. All prices effective to September 1, 2000.

- AHSME (AMC →12) 1989-2000, $1 per copy per year.
- AIME 1989-2000, $2 per copy per year.
- USA and International Math Olympiads, 1989-99, $5 per copy per year.
- National Summary of Results and Awards, 1989-2000, $10 per copy per year.
- Problem Book I, AHSMEs 1950-60, Problem Book II, AHSMEs 1961-65, $10/ea
- Problem Book III, AHSMEs 1966-72, Problem Book IV, AHSMEs 1973-82, $13/ea
- Problem Book V, AHSMEs and AIMEs 1983-88, $28/ea
- USA Mathematical Olympiad Book 1972-86, $18/ea
- International Mathematical Olympiad Book I, 1959-77, $15/ea
- International Mathematical Olympiad Book II, 1978-85, $18/ea
- World Olympiad Problems/Solutions 1995-96, 1996-97, 1997-98, $15/ea
- The Arbelos, Volumes I-V, and a Special Geometry Issue, $8/ea

Shipping & Handling charges for Publication Orders:

Order Total	Add:	Order Total	Add:
$ 10.00 -- $ 30.00	$ 5	$ 40.01 -- $ 50.00	$ 9
$ 30.01 -- $ 40.00	$ 7	$ 50.01 -- $ 75.00	$12
	$ 75.01 -- up	$15	

AMC

2000

AMC →10

☞ DO NOT OPEN UNTIL TUESDAY, FEBRUARY 15, 2000

****Administration On An Earlier Date Will Disqualify Your School's Results****

1. All information (Rules and Instructions) needed to administer this exam is contained in the TEACHERS' MANUAL, which is outside of this package. **PLEASE READ THE MANUAL BEFORE FEBRUARY 15.** Nothing is needed from inside this package until February 15.

2. Your PRINCIPAL or VICE PRINCIPAL must sign the Certification Form A found in the Teachers' Manual.

3. The Answer Forms must be mailed by First Class mail to the AMC Director, Titu Andreescu, no later than 24 hours following the examination.

4. <u>Please Note:</u> All Problems and Solutions are copyrighted; it is illegal to make copies or transmit them on the internet or world wide web without permission.

5. *The publication, reproduction or communication of the problems or solutions of this test during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Duplication at any time via copier, telephone, eMail, World Wide Web or media of any type is a violation of the copyright law.*

HSDISK#4