# Wednesday, FEBRUARY 27, 2002

# Contest B

**The MATHEMATICAL ASSOCIATION OF AMERICA**
**American Mathematics Competitions**
Presented by the Akamai Foundation

# AMC 10

3rd Annual American Mathematics Contest 10

1. DO NOT OPEN THIS BOOKLET UNTIL TOLD TO DO SO BY YOUR PROCTOR.
2. This is a twenty-five question, multiple choice test. Each question is followed by answers marked A,B,C,D and E. Only one of these is correct.
3. The answers to the problems are to be marked on the AMC 10 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. SCORING: You will receive 6 points for each correct answer, 2.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, ruler, compass, protractor, erasers and calculators that are accepted for use on the SAT. No problems on the test will *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form. When your proctor gives the signal, begin working the problems. You will have **75 MINUTES** working time to complete the test.
8. When you finish the exam, *sign your name* in the space provided on the Answer Form.

*Students who score in the top 1% on this AMC 10 will be invited to take the 20th annual American Invitational Mathematics Examination (AIME) on Tuesday, March 26, 2002 or on Tuesday, April 9, 2002. More details about the AIME and other information are on the back page of this test booklet.*

The Committee on the American Mathematics Competitions (CAMC) reserves the right to re-examine students before deciding whether to grant official status to their scores. The CAMC also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

1. The ratio $\dfrac{2^{2001} \cdot 3^{2003}}{6^{2002}}$ is

   **(A)** $\frac{1}{6}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{2}{3}$ **(E)** $\frac{3}{2}$

2. For the nonzero numbers $a$, $b$, and $c$, define

$$(a, b, c) = \frac{abc}{a+b+c}.$$

   Find $(2, 4, 6)$.

   **(A)** 1 **(B)** 2 **(C)** 4 **(D)** 6 **(E)** 24

3. The arithmetic mean of the nine numbers in the set $\{9,99,999,9999,\ldots,999999999\}$ is a 9-digit number $M$, all of whose digits are distinct. The number $M$ does not contain the digit

   **(A)** 0 **(B)** 2 **(C)** 4 **(D)** 6 **(E)** 8

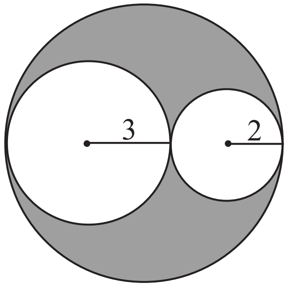
4. What is the value of

$$(3x-2)(4x+1) - (3x-2)4x + 1$$

   when $x = 4$?

   **(A)** 0 **(B)** 1 **(C)** 10 **(D)** 11 **(E)** 12

5. Circles of radius 2 and 3 are externally tangent and are circumscribed by a third circle, as shown in the figure. Find the area of the shaded region.




   **(A)** $3\pi$ **(B)** $4\pi$ **(C)** $6\pi$ **(D)** $9\pi$ **(E)** $12\pi$

6. For how many positive integers $n$ is $n^2 - 3n + 2$ a prime number?

**(A)** none **(B)** one **(C)** two **(D)** more than two, but finitely many **(E)** infinitely many

7. Let $n$ be a positive integer such that $\frac{1}{2} + \frac{1}{3} + \frac{1}{7} + \frac{1}{n}$ is an integer. Which of the following statements is **not** true:

**(A)** 2 divides $n$ **(B)** 3 divides $n$ **(C)** 6 divides $n$ **(D)** 7 divides $n$ **(E)** $n > 84$

8. Suppose July of year $N$ has five Mondays. Which of the following must occur five times in August of year $N$? (Note: Both months have 31 days.)

**(A)** Monday **(B)** Tuesday **(C)** Wednesday **(D)** Thursday **(E)** Friday

9. Using the letters $A$, $M$, $O$, $S$, and $U$, we can form 120 five-letter "words". If these "words" are arranged in alphabetical order, then the "word" *USAMO* occupies position

**(A)** 112 **(B)** 113 **(C)** 114 **(D)** 115 **(E)** 116

10. Suppose that $a$ and $b$ are nonzero real numbers, and that the equation $x^2 + ax + b = 0$ has solutions $a$ and $b$. Then the pair $(a, b)$ is

**(A)** $(-2, 1)$ **(B)** $(-1, 2)$ **(C)** $(1, -2)$ **(D)** $(2, -1)$ **(E)** $(4, 4)$

11. The product of three consecutive positive integers is 8 times their sum. What is the sum of their squares?

**(A)** 50 **(B)** 77 **(C)** 110 **(D)** 149 **(E)** 194

12. For which of the following values of $k$ does the equation $\dfrac{x-1}{x-2} = \dfrac{x-k}{x-6}$ have no solution for $x$?

**(A)** 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

13. Find the value(s) of $x$ such that $8xy - 12y + 2x - 3 = 0$ is true for all values of $y$.

**(A)** $\dfrac{2}{3}$ **(B)** $\dfrac{3}{2}$ or $-\dfrac{1}{4}$ **(C)** $-\dfrac{2}{3}$ or $-\dfrac{1}{4}$ **(D)** $\dfrac{3}{2}$ **(E)** $-\dfrac{3}{2}$ or $-\dfrac{1}{4}$

14. The number $25^{64} \cdot 64^{25}$ is the square of a positive integer $N$. In decimal representation, the sum of the digits of $N$ is

**(A)** 7 **(B)** 14 **(C)** 21 **(D)** 28 **(E)** 35

15. The positive integers $A$, $B$, $A - B$, and $A + B$ are all prime numbers. The sum of these four primes is

**(A)** even **(B)** divisible by 3 **(C)** divisible by 5 **(D)** divisible by 7

**(E)** prime

16. For how many integers $n$ is $\frac{n}{20-n}$ the square of an integer?

**(A)** 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 10

17. A regular octagon $ABCDEFGH$ has sides of length two. Find the area of $\triangle ADG$.

**(A)** $4 + 2\sqrt{2}$ **(B)** $6 + \sqrt{2}$ **(C)** $4 + 3\sqrt{2}$ **(D)** $3 + 4\sqrt{2}$ **(E)** $8 + \sqrt{2}$

18. Four distinct circles are drawn in a plane. What is the maximum number of points where at least two of the circles intersect?

**(A)** 8 **(B)** 9 **(C)** 10 **(D)** 12 **(E)** 16

19. Suppose that $\{a_n\}$ is an arithmetic sequence with

$$a_1 + a_2 + \cdots + a_{100} = 100 \quad \text{and} \quad a_{101} + a_{102} + \cdots + a_{200} = 200.$$

What is the value of $a_2 - a_1$?

**(A)** 0.0001 **(B)** 0.001 **(C)** 0.01 **(D)** 0.1 **(E)** 1

20. Let $a, b$, and $c$ be real numbers such that $a - 7b + 8c = 4$ and $8a + 4b - c = 7$. Then $a^2 - b^2 + c^2$ is

**(A)** 0 **(B)** 1 **(C)** 4 **(D)** 7 **(E)** 8

21. Andy's lawn has twice as much area as Beth's lawn and three times as much area as Carlos' lawn. Carlos' lawn mower cuts half as fast as Beth's mower and one third as fast as Andy's mower. If they all start to mow their lawns at the same time, who will finish first?

**(A)** Andy **(B)** Beth **(C)** Carlos **(D)** Andy and Carlos tie for first.

**(E)** All three tie.

22. Let $\triangle XOY$ be a right-angled triangle with $m\angle XOY = 90°$. Let $M$ and $N$ be the midpoints of legs $OX$ and $OY$, respectively. Given that $XN = 19$ and $YM = 22$, find $XY$.

**(A)** 24 **(B)** 26 **(C)** 28 **(D)** 30 **(E)** 32

23. Let $\{a_k\}$ be a sequence of integers such that $a_1 = 1$ and $a_{m+n} = a_m + a_n + mn$, for all positive integers $m$ and $n$. Then $a_{12}$ is

**(A)** 45 **(B)** 56 **(C)** 67 **(D)** 78 **(E)** 89

24. Riders on a Ferris wheel travel in a circle in a vertical plane. A particular wheel has radius 20 feet and revolves at the constant rate of one revolution per minute. How many seconds does it take a rider to travel from the bottom of the wheel to a point 10 vertical feet above the bottom?

**(A)** 5 **(B)** 6 **(C)** 7.5 **(D)** 10 **(E)** 15

25. When 15 is appended to a list of integers, the mean is increased by 2. When 1 is appended to the enlarged list, the mean of the enlarged list is decreased by 1. How many integers were in the original list?

**(A)** 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

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## WRITE TO US!

*Correspondence about the problems and solutions for this **AMC 10** should be addressed to:*


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## 2002 AIME

The AIME will be held on Tuesday, March 26, 2002 with the alternate on April 9,2002. It is a 15-question, 3-hour, integer-answer exam. You will be invited to participate only if you score in the top 1% of this AMC 10 or receive a score of 100 or above on the AMC 12. Alternately, you must be in the top 5% of the AMC 12. Top-scoring students on the AMC 10/12/AIME will be selected to take the USA Mathematical Olympiad (USAMO) on Thursday through Sunday, May 9-12, 2002 in Cambridge, MA. The best way to prepare for the AIME and USAMO is to study previous years of these exams. Copies may be ordered as indicated below.

## PUBLICATIONS

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**Examinations:** Each price is for one copy of an exam and its solutions for one year. Specify the years you want and how many copies of each. All prices effective to September 1, 2002.

- AMC 10 2000-2002/AHSME (AMC 12) 1989-2002, $1 per copy per year.
- AIME 1989-2002, $2 per copy per year.
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2002

# AMC 10 - Contest B

## DO NOT OPEN UNTIL

## Wednesday, FEBRUARY 27, 2002

****Administration On An Earlier Date Will Disqualify Your School's Results****

1. All information (Rules and Instructions) needed to administer this exam is contained in the TEACHERS' MANUAL, which is outside of this package. **PLEASE READ THE MANUAL BEFORE FEBRUARY 27.** Nothing is needed from inside this package until February 27.
2. Your PRINCIPAL or VICE PRINCIPAL must sign the Certification Form A found in the Teachers' Manual.
3. The Answer Forms must be mailed by First Class mail to the AMC Director, Titu Andreescu, no later than 24 hours following the examination.
4. Please Note: All Problems and Solutions are copyrighted; it is illegal to make copies or transmit them on the internet or world wide web without permission.
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