Tuesday, FEBRUARY10, 2004

5th Annual American Mathematics Contest 10

AMC 10

Contest A

The MATHEMATICAL ASSOCIATION OF AMERICA
American Mathematics Competitions

1. DO NOT OPEN THIS BOOKLET UNTIL TOLD TO DO SO BY YOUR PROCTOR.
2. This is a twenty-five question, multiple choice test. Each question is followed by answers marked A,B,C,D and E. Only one of these is correct.
3. The answers to the problems are to be marked on the AMC 10 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. SCORING: You will receive 6 points for each correct answer, 2.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, ruler, compass, protractor, erasers and calculators that are accepted for use on the SAT. No problems on the test will *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form. When your proctor gives the signal, begin working the problems. You will have **75 MINUTES** working time to complete the test.
8. When you finish the exam, *sign your name* in the space provided on the Answer Form.

Students who score 120 or above or finish in the top 1% on this AMC 10 will be invited to take the 22nd annual American Invitational Mathematics Examination (AIME) on Tuesday, March 23, 2004 or on Tuesday, April 6, 2004. More details about the AIME and other information are on the back page of this test booklet.

The Committee on the American Mathematics Competitions (CAMC) reserves the right to re-examine students before deciding whether to grant official status to their scores. The CAMC also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

1. You and five friends need to raise \$1500 in donations for a charity, dividing the fundraising equally. How many dollars will each of you need to raise?

(A) 250 **(B)** 300 **(C)** 1500 **(D)** 7500 **(E)** 9000

2. For any three real numbers a, b, and c, with $b \neq c$, the operation ¶ is defined by

$$¶(a,b,c) = \frac{a}{b-c}.$$

What is $¶\,(¶(1,2,3), ¶(2,3,1), ¶(3,1,2))$?

(A) $-\frac{1}{2}$ **(B)** $-\frac{1}{4}$ **(C)** 0 **(D)** $\frac{1}{4}$ **(E)** $\frac{1}{2}$

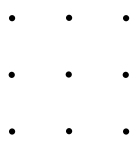
3. Alicia earns \$20 per hour, of which 1.45% is deducted to pay local taxes. How many cents per hour of Alicia's wages are used to pay local taxes?

(A) 0.0029 **(B)** 0.029 **(C)** 0.29 **(D)** 2.9 **(E)** 29

4. What is the value of x if $|x-1| = |x-2|$?

(A) $-\frac{1}{2}$ **(B)** $\frac{1}{2}$ **(C)** 1 **(D)** $\frac{3}{2}$ **(E)** 2

5. A set of three points is chosen randomly from the grid shown. Each three-point set has the same probability of being chosen. What is the probability that the points lie on the same straight line?



(A) $\frac{1}{21}$ **(B)** $\frac{1}{14}$ **(C)** $\frac{2}{21}$ **(D)** $\frac{1}{7}$ **(E)** $\frac{2}{7}$

6. Bertha has 6 daughters and no sons. Some of her daughters have 6 daughters, and the rest have none. Bertha has a total of 30 daughters and granddaughters, and no great-granddaughters. How many of Bertha's daughters and granddaughters have no daughters?

(A) 22 **(B)** 23 **(C)** 24 **(D)** 25 **(E)** 26

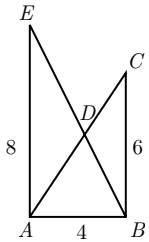
7. A grocer stacks oranges in a pyramid-like stack whose rectangular base is 5 oranges by 8 oranges. Each orange above the first level rests in a pocket formed by four oranges in the level below. The stack is completed by a single row of oranges. How many oranges are in the stack?

(A) 96 **(B)** 98 **(C)** 100 **(D)** 101 **(E)** 134

8. A game is played with tokens according to the following rule. In each round, the player with the most tokens gives one token to each of the other players and also places one token into a discard pile. The game ends when some player runs out of tokens. Players A, B, and C start with 15, 14, and 13 tokens, respectively. How many rounds will there be in the game?

(A) 36 **(B)** 37 **(C)** 38 **(D)** 39 **(E)** 40

9. In the Figure, $\angle EAB$ and $\angle ABC$ are right angles, $AB = 4$, $BC = 6$, $AE = 8$, and $\overline{AC}$ and $\overline{BE}$ intersect at D. What is the difference between the areas of $\triangle ADE$ and $\triangle BDC$?



(A) 2 **(B)** 4 **(C)** 5 **(D)** 8 **(E)** 9

10. Coin A is flipped three times and coin B is flipped four times. What is the probability that the number of heads obtained from flipping the two fair coins is the same?

(A) $\frac{19}{128}$ **(B)** $\frac{23}{128}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{35}{128}$ **(E)** $\frac{1}{2}$

11. A company sells peanut butter in cylindrical jars. Marketing research suggests that using wider jars will increase sales. If the diameter of the jars is increased by 25% without altering the volume, by what percent must the height be decreased?

(A) 10 **(B)** 25 **(C)** 36 **(D)** 50 **(E)** 60

12. Henry's Hamburger Heaven offers its hamburgers with the following condiments: ketchup, mustard, mayonnaise, tomato, lettuce, pickles, cheese, and onions. A customer can choose one, two, or three meat patties, and any collection of condiments. How many different kinds of hamburgers can be ordered?

(A) 24 **(B)** 256 **(C)** 768 **(D)** 40,320 **(E)** 120,960

13. At a party, each man danced with exactly three women and each woman danced with exactly two men. Twelve men attended the party. How many women attended the party?

(A) 8 **(B)** 12 **(C)** 16 **(D)** 18 **(E)** 24

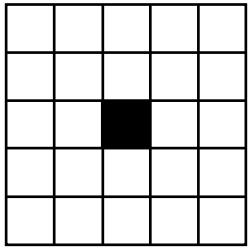
14. The average value of all the pennies, nickels, dimes, and quarters in Paula's purse is 20 cents. If she had one more quarter, the average value would be 21 cents. How many dimes does she have in her purse?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

15. Given that $-4 \leq x \leq -2$ and $2 \leq y \leq 4$, what is the largest possible value of $(x+y)/x$?

(A) -1 **(B)** $-\frac{1}{2}$ **(C)** 0 **(D)** $\frac{1}{2}$ **(E)** 1

16. The 5×5 grid shown contains a collection of squares with sizes from 1×1 to 5×5. How many of these squares contain the black center square?



(A) 12 **(B)** 15 **(C)** 17 **(D)** 19 **(E)** 20

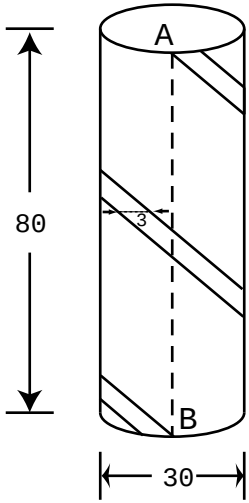
17. Brenda and Sally run in opposite directions on a circular track, starting at diametrically opposite points. They first meet after Brenda has run 100 meters. They next meet after Sally has run 150 meters past their first meeting point. Each girl runs at a constant speed. What is the length of the track in meters?

(A) 250 **(B)** 300 **(C)** 350 **(D)** 400 **(E)** 500

18. A sequence of three real numbers forms an arithmetic progression with a first term of 9. If 2 is added to the second term and 20 is added to the third term, the three resulting numbers form a geometric progression. What is the smallest possible value for the third term of the geometric progression?

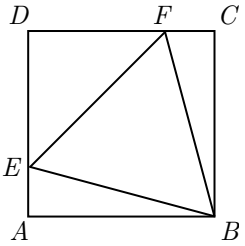
(A) 1 **(B)** 4 **(C)** 36 **(D)** 49 **(E)** 81

19. A white cylindrical silo has a diameter of 30 feet and a height of 80 feet. A red stripe with a horizontal width of 3 feet is painted on the silo, as shown, making two complete revolutions around it. What is the area of the stripe in square feet?



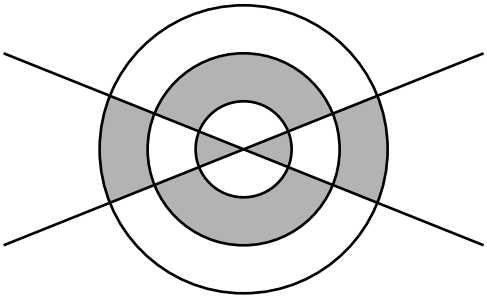
(A) 120 **(B)** 180 **(C)** 240 **(D)** 360 **(E)** 480

20. Points E and F are located on square $ABCD$ so that $\triangle BEF$ is equilateral. What is the ratio of the area of $\triangle DEF$ to that of $\triangle ABE$?



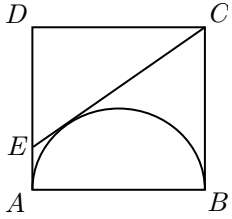
(A) $\frac{4}{3}$ **(B)** $\frac{3}{2}$ **(C)** $\sqrt{3}$ **(D)** 2 **(E)** $1+\sqrt{3}$

21. Two distinct lines pass through the center of three concentric circles of radii 3, 2, and 1. The area of the shaded region in the diagram is 8/13 of the area of the unshaded region. What is the radian measure of the acute angle formed by the two lines? (Note: π radians is 180 degrees.)



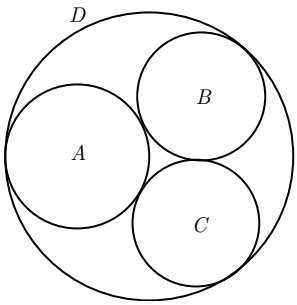
(A) $\frac{\pi}{8}$ **(B)** $\frac{\pi}{7}$ **(C)** $\frac{\pi}{6}$ **(D)** $\frac{\pi}{5}$ **(E)** $\frac{\pi}{4}$

22. Square $ABCD$ has side length 2. A semicircle with diameter $\overline{AB}$ is constructed inside the square, and the tangent to the semicircle from C intersects side $\overline{AD}$ at E. What is the length of $\overline{CE}$?



(A) $\frac{2+\sqrt{5}}{2}$ **(B)** $\sqrt{5}$ **(C)** $\sqrt{6}$ **(D)** $\frac{5}{2}$ **(E)** $5-\sqrt{5}$

23. Circles A, B, and C are externally tangent to each other and internally tangent to circle D. Circles B and C are congruent. Circle A has radius 1 and passes through the center of D. What is the radius of circle B?



(A) $\frac{2}{3}$ **(B)** $\frac{\sqrt{3}}{2}$ **(C)** $\frac{7}{8}$ **(D)** $\frac{8}{9}$ **(E)** $\frac{1+\sqrt{3}}{3}$

24. Let $a_1, a_2, \cdots$, be a sequence with the following properties.

    (i) $a_1 = 1$, and

    (ii) $a_{2n} = n \cdot a_n$ for any positive integer n.

    What is the value of $a_{2^{100}}$?

    (A) 1 **(B)** 2^{99} **(C)** 2^{100} **(D)** 2^{4950} **(E)** 2^{9999}

25. Three mutually tangent spheres of radius 1 rest on a horizontal plane. A sphere of radius 2 rests on them. What is the distance from the plane to the top of the larger sphere?

    (A) $3 + \frac{\sqrt{30}}{2}$ **(B)** $3 + \frac{\sqrt{69}}{3}$ **(C)** $3 + \frac{\sqrt{123}}{4}$ **(D)** $\frac{52}{9}$ **(E)** $3 + 2\sqrt{2}$

WRITE TO US!

*Correspondence about the problems and solutions for this **AMC 10** should be addressed to:*

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Orders for any of the publications listed below should be addressed to:

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2004 AIME

The AIME will be held on Tuesday, March 23, 2004 with the alternate on April 6, 2004. It is a 15-question, 3-hour, integer-answer exam. You will be invited to participate only if you score 120 or above or finish in the top 1% of the AMC 10 or receive a score of 100 or above on the AMC 12. Alternately, you must be in the top 5% of the AMC 12. Top-scoring students on the AMC 10/12/AIME will be selected to take the USA Mathematical Olympiad (USAMO) in late Spring. The best way to prepare for the AIME and USAMO is to study previous years of these exams. Copies may be ordered as indicated below.

PUBLICATIONS

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- AMC 10 2000-2004/(AHSME) AMC 12 1989-2004, $1 per exam copy.
- AIME 1989-2004, $2 per copy per year (2004 available after April).
- USA and International Math Olympiads, 1989-1999, $5 per copy per year, 2000-$14, 2001-$17
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2004

AMC 10 - Contest A

DO NOT OPEN UNTIL TUESDAY, FEBRUARY 10, 2004

****Administration On An Earlier Date Will Disqualify Your School's Results****

1. All information (Rules and Instructions) needed to administer this exam is contained in the TEACHER'S MANUAL, which is outside of this package. **PLEASE READ THE MANUAL BEFORE FEBRUARY 10.** Nothing is needed from inside this package until February 10.
2. Your PRINCIPAL or VICE PRINCIPAL must sign the Certification Form A found in the Teachers' Manual.
3. The Answer Forms must be mailed by First Class mail to the AMC no later than 24 hours following the examination.
4. Please Note: All Problems and Solutions are copyrighted; it is illegal to make copies or transmit them on the internet or world wide web without permission.
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