Wednesday, FEBRUARY 25, 2004

5th Annual American Mathematics Contest 10

AMC 10

Contest B

The MATHEMATICAL ASSOCIATION OF AMERICA
American Mathematics Competitions

1. DO NOT OPEN THIS BOOKLET UNTIL TOLD TO DO SO BY YOUR PROCTOR.
2. This is a twenty-five question, multiple choice test. Each question is followed by answers marked A,B,C,D and E. Only one of these is correct.
3. The answers to the problems are to be marked on the AMC 10 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. SCORING: You will receive 6 points for each correct answer, 2.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, ruler, compass, protractor, erasers and calculators that are accepted for use on the SAT. No problems on the test will *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form. When your proctor gives the signal, begin working the problems. You will have **75 MINUTES** working time to complete the test.
8. When you finish the exam, *sign your name* in the space provided on the Answer Form.

Students who score 120 or above or finish in the top 1% on this AMC 10 will be invited to take the 22nd annual American Invitational Mathematics Examination (AIME) on Tuesday, March 23, 2004 or on Tuesday, April 6, 2004. More details about the AIME and other information are on the back page of this test booklet.

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1. Each row of the Misty Moon Amphitheater has 33 seats. Rows 12 through 22 are reserved for a youth club. How many seats are reserved for this club?

   (A) 297 **(B)** 330 **(C)** 363 **(D)** 396 **(E)** 726

2. How many two-digit positive integers have at least one 7 as a digit?

   (A) 10 **(B)** 18 **(C)** 19 **(D)** 20 **(E)** 30

3. At each basketball practice last week, Jenny made twice as many free throws as she made at the previous practice. At her fifth practice she made 48 free throws. How many free throws did she make at the first practice?

   (A) 3 **(B)** 6 **(C)** 9 **(D)** 12 **(E)** 15

4. A standard six-sided die is rolled, and P is the product of the five numbers that are visible. What is the largest number that is certain to divide P?

   (A) 6 **(B)** 12 **(C)** 24 **(D)** 144 **(E)** 720

5. In the expression $c \cdot a^b - d$, the values of a, b, c, and d are 0, 1, 2, and 3, although not necessarily in that order. What is the maximum possible value of the result?

   (A) 5 **(B)** 6 **(C)** 8 **(D)** 9 **(E)** 10

6. Which of the following numbers is a perfect square?

   (A) $98! \cdot 99!$ **(B)** $98! \cdot 100!$ **(C)** $99! \cdot 100!$ **(D)** $99! \cdot 101!$ **(E)** $100! \cdot 101!$

7. On a trip from the United States to Canada, Isabella took d U.S. dollars. At the border she exchanged them all, receiving 10 Canadian dollars for every 7 U.S. dollars. After spending 60 Canadian dollars, she had d Canadian dollars left. What is the sum of the digits of d?

   (A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

8. Minneapolis-St. Paul International Airport is 8 miles southwest of downtown St. Paul and 10 miles southeast of downtown Minneapolis. Which of the following is closest to the number of miles between downtown St. Paul and downtown Minneapolis?

   (A) 13 **(B)** 14 **(C)** 15 **(D)** 16 **(E)** 17

9. A square has sides of length 10, and a circle centered at one of its vertices has radius 10. What is the area of the union of the regions enclosed by the square and the circle?

   (A) $200 + 25\pi$ **(B)** $100 + 75\pi$ **(C)** $75 + 100\pi$ **(D)** $100 + 100\pi$ **(E)** $100 + 125\pi$

10. A grocer makes a display of cans in which the top row has one can and each lower row has two more cans than the row above it. If the display contains 100 cans, how many rows does it contain?

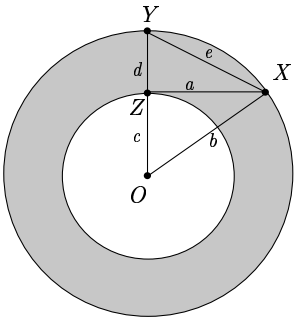
    (A) 5 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 11

11. Two eight-sided dice each have faces numbered 1 through 8. When the dice are rolled, each face has an equal probability of appearing on the top. What is the probability that the product of the two top numbers is greater than their sum?

(A) $\frac{1}{2}$ **(B)** $\frac{47}{64}$ **(C)** $\frac{3}{4}$ **(D)** $\frac{55}{64}$ **(E)** $\frac{7}{8}$

12. An *annulus* is the region between two concentric circles. The concentric circles in the figure have radii b and c, with $b > c$. Let $\overline{OX}$ be a radius of the larger circle, let $\overline{XZ}$ be tangent to the smaller circle at Z, and let $\overline{OY}$ be the radius of the larger circle that contains Z. Let $a = XZ$, $d = YZ$, and $e = XY$. What is the area of the annulus?

(A) πa^2 **(B)** πb^2 **(C)** πc^2 **(D)** πd^2 **(E)** πe^2



13. In the United States, coins have the following thicknesses: penny, 1.55 mm; nickel, 1.95 mm; dime, 1.35 mm; quarter, 1.75 mm. If a stack of these coins is exactly 14 mm high, how many coins are in the stack?

(A) 7 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 11

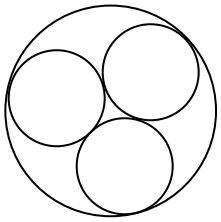
14. A bag initially contains red marbles and blue marbles only, with more blue than red. Red marbles are added to the bag until only 1/3 of the marbles in the bag are blue. Then yellow marbles are added to the bag until only 1/5 of the marbles in the bag are blue. Finally, the number of blue marbles in the bag is doubled. What fraction of the marbles now in the bag are blue?

(A) $\frac{1}{5}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{1}{3}$ **(D)** $\frac{2}{5}$ **(E)** $\frac{1}{2}$

15. Patty has 20 coins consisting of nickels and dimes. If her nickels were dimes and her dimes were nickels, she would have 70 cents more. How much are her coins worth?

(A) \$1.15 **(B)** \$1.20 **(C)** \$1.25 **(D)** \$1.30 **(E)** \$1.35

16. Three circles of radius 1 are externally tangent to each other and internally tangent to a larger circle. What is the radius of the large circle?

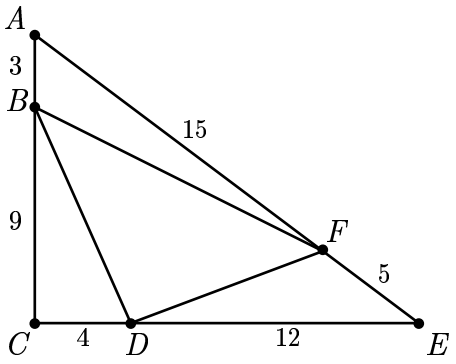


(A) $\frac{2+\sqrt{6}}{3}$ **(B)** 2 **(C)** $\frac{2+3\sqrt{2}}{3}$ **(D)** $\frac{3+2\sqrt{3}}{3}$ **(E)** $\frac{3+\sqrt{3}}{2}$

17. The two digits in Jack's age are the same as the digits in Bill's age, but in reverse order. In five years Jack will be twice as old as Bill will be then. What is the difference in their current ages?

(A) 9 **(B)** 18 **(C)** 27 **(D)** 36 **(E)** 45

18. In right triangle $\triangle ACE$, we have $AC = 12$, $CE = 16$, and $EA = 20$. Points B, D, and F are located on $\overline{AC}$, $\overline{CE}$, and $\overline{EA}$, respectively, so that $AB = 3$, $CD = 4$, and $EF = 5$. What is the ratio of the area of $\triangle BDF$ to that of $\triangle ACE$?

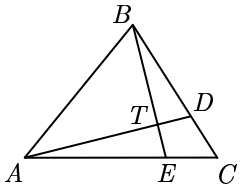


(A) $\frac{1}{4}$ **(B)** $\frac{9}{25}$ **(C)** $\frac{3}{8}$ **(D)** $\frac{11}{25}$ **(E)** $\frac{7}{16}$

19. In the sequence 2001, 2002, 2003, ..., each term after the third is found by subtracting the previous term from the sum of the two terms that precede that term. For example, the fourth term is $2001 + 2002 - 2003 = 2000$. What is the 2004^{th} term in this sequence?

(A) -2004 **(B)** -2 **(C)** 0 **(D)** 4003 **(E)** 6007

20. In $\triangle ABC$ points D and E lie on $\overline{BC}$ and $\overline{AC}$, respectively. If $\overline{AD}$ and $\overline{BE}$ intersect at T so that $AT/DT = 3$ and $BT/ET = 4$, what is CD/BD?



(A) $\frac{1}{8}$ **(B)** $\frac{2}{9}$ **(C)** $\frac{3}{10}$ **(D)** $\frac{4}{11}$ **(E)** $\frac{5}{12}$

21. Let $1, 4, \ldots$ and $9, 16, \ldots$ be two arithmetic progressions. The set S is the union of the first 2004 terms of each sequence. How many distinct numbers are in S?

(A) 3722 **(B)** 3732 **(C)** 3914 **(D)** 3924 **(E)** 4007

22. A triangle with sides of 5, 12, and 13 has both an inscribed and a circumscribed circle. What is the distance between the centers of those circles?

(A) $\frac{3\sqrt{5}}{2}$ **(B)** $\frac{7}{2}$ **(C)** $\sqrt{15}$ **(D)** $\frac{\sqrt{65}}{2}$ **(E)** $\frac{9}{2}$

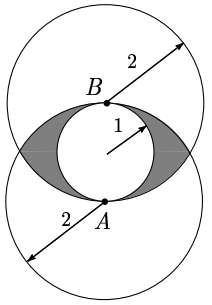
23. Each face of a cube is painted either red or blue, each with probability 1/2. The color of each face is determined independently. What is the probability that the painted cube can be placed on a horizontal surface so that the four vertical faces are all the same color?

(A) $\frac{1}{4}$ **(B)** $\frac{5}{16}$ **(C)** $\frac{3}{8}$ **(D)** $\frac{7}{16}$ **(E)** $\frac{1}{2}$

24. In $\triangle ABC$ we have $AB = 7$, $AC = 8$, and $BC = 9$. Point D is on the circumscribed circle of the triangle so that $\overline{AD}$ bisects $\angle BAC$. What is the value of AD/CD?

(A) $\frac{9}{8}$ **(B)** $\frac{5}{3}$ **(C)** 2 **(D)** $\frac{17}{7}$ **(E)** $\frac{5}{2}$

25. A circle of radius 1 is internally tangent to two circles of radius 2 at points A and B, where AB is a diameter of the smaller circle. What is the area of the region, shaded in the figure, that is outside the smaller circle and inside each of the two larger circles?



(A) $\frac{5}{3}\pi - 3\sqrt{2}$ **(B)** $\frac{5}{3}\pi - 2\sqrt{3}$ **(C)** $\frac{8}{3}\pi - 3\sqrt{3}$ **(D)** $\frac{8}{3}\pi - 3\sqrt{2}$

(E) $\frac{8}{3}\pi - 2\sqrt{3}$

WRITE TO US!

*Correspondence about the problems and solutions for this **AMC 10** should be addressed to:*

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Orders for any of the publications listed below should be addressed to:

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2004 AIME

The AIME will be held on Tuesday, March 23, 2004 with the alternate on April 6, 2004. It is a 15-question, 3-hour, integer-answer exam. You will be invited to participate only if you score 120 or above or finish in the top 1% of the AMC 10 or receive a score of 100 or above on the AMC 12. Alternately, you must be in the top 5% of the AMC 12. Top-scoring students on the AMC 10/12/AIME will be selected to take the USA Mathematical Olympiad (USAMO) in late Spring. The best way to prepare for the AIME and USAMO is to study previous years of these exams. Copies may be ordered as indicated below.

PUBLICATIONS

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- AMC 10 2000-2004/(AHSME) AMC 12 1989-2004, $1 per exam copy.
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2004

AMC 10 - Contest B

DO NOT OPEN UNTIL
Wednesday, FEBRUARY 25, 2004

****Administration On An Earlier Date Will Disqualify Your School's Results****

1. All information (Rules and Instructions) needed to administer this exam is contained in the TEACHER'S MANUAL, which is outside of this package. **PLEASE READ THE MANUAL BEFORE FEBRUARY 25.** Nothing is needed from inside this package until February 25.
2. Your PRINCIPAL or VICE PRINCIPAL must sign the Certification Form A found in the Teachers' Manual.
3. The Answer Forms must be mailed by First Class mail to the AMC no later than 24 hours following the examination.
4. Please Note: All Problems and Solutions are copyrighted; it is illegal to make copies or transmit them on the internet or world wide web without permission.
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