Wednesday, FEBRUARY 16, 2005

6th Annual American Mathematics Contest 10

AMC 10

Contest B

The MATHEMATICAL ASSOCIATION OF AMERICA
American Mathematics Competitions

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR GIVES THE SIGNAL TO BEGIN.
2. This is a 25-question, multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Mark your answer to each problem on the AMC 10 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. SCORING: You will receive 6 points for each correct answer, 2.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, ruler, compass, protractor, erasers and calculators that are accepted for use on the SAT. No problems on the test will *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form. When your proctor gives the signal, begin working the problems. You will have **75 MINUTES** to complete the test.
8. When you finish the exam, *sign your name* in the space provided on the Answer Form.

Students who score 120 or above or finish in the top 1% on this AMC 10 will be invited to take the 23rd annual American Invitational Mathematics Examination (AIME) on Tuesday, March 8, 2005 or Tuesday, March 22, 2005. More details about the AIME and other information are on the back page of this test booklet.

The Committee on the American Mathematics Competitions (CAMC) reserves the right to re-examine students before deciding whether to grant official status to their scores. The CAMC also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

The publication, reproduction or communication of the problems or solutions of the AMC 10 during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination at any time via copier, telephone, email, World Wide Web or media of any type is a violation of the competition rules.

1. A scout troop buys 1000 candy bars at a price of five for \$2. They sell all the candy bars at a price of two for \$1. What was their profit, in dollars?

(A) 100 **(B)** 200 **(C)** 300 **(D)** 400 **(E)** 500

2. A positive number x has the property that $x\%$ of x is 4. What is x?

(A) 2 **(B)** 4 **(C)** 10 **(D)** 20 **(E)** 40

3. A gallon of paint is used to paint a room. One third of the paint is used on the first day. On the second day, one third of the remaining paint is used. What fraction of the original amount of paint is available to use on the third day?

(A) $\frac{1}{10}$ **(B)** $\frac{1}{9}$ **(C)** $\frac{1}{3}$ **(D)** $\frac{4}{9}$ **(E)** $\frac{5}{9}$

4. For real numbers a and b, define $a \diamond b = \sqrt{a^2 + b^2}$. What is the value of

$$(5 \diamond 12) \diamond ((-12) \diamond (-5)) \ ?$$

(A) 0 **(B)** $\frac{17}{2}$ **(C)** 13 **(D)** $13\sqrt{2}$ **(E)** 26

5. Brianna is using part of the money she earned on her weekend job to buy several equally-priced CDs. She used one fifth of her money to buy one third of the CDs. What fraction of her money will she have left after she buys all the CDs?

(A) $\frac{1}{5}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{2}{5}$ **(D)** $\frac{2}{3}$ **(E)** $\frac{4}{5}$

6. At the beginning of the school year, Lisa's goal was to earn an A on at least 80% of her 50 quizzes for the year. She earned an A on 22 of the first 30 quizzes. If she is to achieve her goal, on at most how many of the remaining quizzes can she earn a grade lower than an A?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

7. A circle is inscribed in a square, then a square is inscribed in this circle, and finally, a circle is inscribed in this square. What is the ratio of the area of the smaller circle to the area of the larger square?

(A) $\frac{\pi}{16}$ **(B)** $\frac{\pi}{8}$ **(C)** $\frac{3\pi}{16}$ **(D)** $\frac{\pi}{4}$ **(E)** $\frac{\pi}{2}$

8. An 8-foot by 10-foot floor is tiled with square tiles of size 1 foot by 1 foot. Each tile has a pattern consisting of four white quarter circles of radius 1/2 foot centered at each corner of the tile. The remaining portion of the tile is shaded. How many square feet of the floor are shaded?

(A) $80 - 20\pi$ **(B)** $60 - 10\pi$ **(C)** $80 - 10\pi$ **(D)** $60 + 10\pi$ **(E)** $80 + 10\pi$

9. One fair die has faces 1, 1, 2, 2, 3, 3 and another has faces 4, 4, 5, 5, 6, 6. The dice are rolled and the numbers on the top faces are added. What is the probability that the sum will be odd?

(A) $\frac{1}{3}$ **(B)** $\frac{4}{9}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{5}{9}$ **(E)** $\frac{2}{3}$

10. In $\triangle ABC$, we have $AC = BC = 7$ and $AB = 2$. Suppose that D is a point on line AB such that B lies between A and D and $CD = 8$. What is BD?

(A) 3 **(B)** $2\sqrt{3}$ **(C)** 4 **(D)** 5 **(E)** $4\sqrt{2}$

11. The first term of a sequence is 2005. Each succeeding term is the sum of the cubes of the digits of the previous term. What is the 2005^{th} term of the sequence?

(A) 29 **(B)** 55 **(C)** 85 **(D)** 133 **(E)** 250

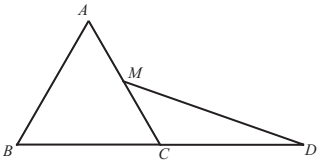
12. Twelve fair dice are rolled. What is the probability that the **product** of the numbers on the top faces is prime?

(A) $\left(\frac{1}{12}\right)^{12}$ **(B)** $\left(\frac{1}{6}\right)^{12}$ **(C)** $2\left(\frac{1}{6}\right)^{11}$ **(D)** $\frac{5}{2}\left(\frac{1}{6}\right)^{11}$ **(E)** $\left(\frac{1}{6}\right)^{10}$

13. How many numbers between 1 and 2005 are integer multiples of 3 or 4 but not 12?

(A) 501 **(B)** 668 **(C)** 835 **(D)** 1002 **(E)** 1169

14. Equilateral $\triangle ABC$ has side length 2, M is the midpoint of $\overline{AC}$, and C is the midpoint of $\overline{BD}$. What is the area of $\triangle CDM$?



(A) $\frac{\sqrt{2}}{2}$ **(B)** $\frac{3}{4}$ **(C)** $\frac{\sqrt{3}}{2}$ **(D)** 1 **(E)** $\sqrt{2}$

15. An envelope contains eight bills: 2 ones, 2 fives, 2 tens, and 2 twenties. Two bills are drawn at random without replacement. What is the probability that their sum is \$20 or more?

(A) $\frac{1}{4}$ **(B)** $\frac{2}{5}$ **(C)** $\frac{3}{7}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{2}{3}$

16. The quadratic equation $x^2 + mx + n = 0$ has roots that are twice those of $x^2 + px + m = 0$, and none of m, n, and p is zero. What is the value of n/p?

(A) 1 **(B)** 2 **(C)** 4 **(D)** 8 **(E)** 16

17. Suppose that $4^a = 5$, $5^b = 6$, $6^c = 7$, and $7^d = 8$. What is $a \cdot b \cdot c \cdot d$?

(A) 1 **(B)** $\frac{3}{2}$ **(C)** 2 **(D)** $\frac{5}{2}$ **(E)** 3

18. All of David's telephone numbers have the form 555–*abc*–*defg*, where a, b, c, d, e, f, and g are distinct digits and in increasing order, and none is either 0 or 1. How many different telephone numbers can David have?

(A) 1 **(B)** 2 **(C)** 7 **(D)** 8 **(E)** 9

19. On a certain math exam, 10% of the students got 70 points, 25% got 80 points, 20% got 85 points, 15% got 90 points, and the rest got 95 points. What is the difference between the mean and the median score on this exam?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 4 **(E)** 5

20. What is the average (mean) of all 5-digit numbers that can be formed by using each of the digits 1, 3, 5, 7, and 8 exactly once?

(A) 48000 **(B)** 49999.5 **(C)** 53332.8 **(D)** 55555 **(E)** 56432.8

21. Forty slips are placed into a hat, each bearing a number 1, 2, 3, 4, 5, 6, 7, 8, 9, or 10, with each number entered on four slips. Four slips are drawn from the hat at random and without replacement. Let p be the probability that all four slips bear the same number. Let q be the probability that two of the slips bear a number a and the other two bear a number $b \neq a$. What is the value of q/p?

(A) 162 **(B)** 180 **(C)** 324 **(D)** 360 **(E)** 720

22. For how many positive integers n less than or equal to 24 is $n!$ evenly divisible by $1 + 2 + \cdots + n$?

(A) 8 **(B)** 12 **(C)** 16 **(D)** 17 **(E)** 21

23. In trapezoid $ABCD$ we have $\overline{AB}$ parallel to $\overline{DC}$, E as the midpoint of $\overline{BC}$, and F as the midpoint of $\overline{DA}$. The area of $ABEF$ is twice the area of $FECD$. What is AB/DC?

(A) 2 **(B)** 3 **(C)** 5 **(D)** 6 **(E)** 8

24. Let x and y be two-digit integers such that y is obtained by reversing the digits of x. The integers x and y satisfy $x^2 - y^2 = m^2$ for some positive integer m. What is $x + y + m$?

(A) 88 **(B)** 112 **(C)** 116 **(D)** 144 **(E)** 154

25. A subset B of the set of integers from 1 to 100, inclusive, has the property that no two elements of B sum to 125. What is the maximum possible number of elements in B?

(A) 50 **(B)** 51 **(C)** 62 **(D)** 65 **(E)** 68

WRITE TO US!

Correspondence about the problems and solutions for this AMC 10 and orders for any of the publications listed below should be addressed to:

American Mathematics Competitions
University of Nebraska, P.O. Box 81606
Lincoln, NE 68501-1606
Phone: 402-472-2257; Fax: 402-472-6087; email: amcinfo@unl.edu

The problems and solutions for this AMC 10 were prepared by the MAA's Committee on the AMC 10 and AMC 12 under the direction of AMC 10 Subcommittee Chair:

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2005 AIME

The AIME will be held on Tuesday, March 8, 2005 with the alternate on March 22, 2005. It is a 15-question, 3-hour, integer-answer exam. You will be invited to participate only if you score 120 or above, or finish in the top 1% of the AMC 10, or if you score 100 or above or finish in the top 5% of the AMC 12. Top-scoring students on the AMC 10/12/AIME will be selected to take the USA Mathematical Olympiad (USAMO) on April 19 and 20, 2005. The best way to prepare for the AIME and USAMO is to study previous exams. Copies may be ordered as indicated below.

PUBLICATIONS

MINIMUM ORDER: $10 (before shipping/handling fee), *PAYMENT IN U.S. FUNDS ONLY made payable to the* American Mathematics Competitions or VISA/MASTERCARD accepted. Include card number, expiration date, cardholder name, address, telephone and email. U.S.A. and Canadian orders must be prepaid and will be shipped Priority Mail, UPS or Air Mail.
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Examinations: Each price is for one copy of an exam and its solutions. Specify the years you want and how many copies of each. All prices effective to September 1, 2005.

- AMC 10 2000–2005/(AHSME) AMC 12 1989–2005, $1 per exam copy.
- AIME 1983–1993, 1995–2005, $2 per copy per year (2005 available after March).
- USA and International Math Olympiads, 1989–1999, $5 per copy per year, (quantities limited)
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2005

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DO NOT OPEN UNTIL WEDNESDAY, FEBRUARY 16, 2005

****Administration On An Earlier Date Will Disqualify Your School's Results****

1. All information (Rules and Instructions) needed to administer this exam is contained in the TEACHERS' MANUAL, which is outside of this package. **PLEASE READ THE MANUAL BEFORE FEBRUARY 16.** Nothing is needed from inside this package until February 16.
2. Your PRINCIPAL or VICE PRINCIPAL must sign the Certification Form found in the Teachers' Manual.
3. The Answer Forms must be mailed by First Class mail to the AMC no later than 24 hours following the examination.
4. *The publication, reproduction or communication of the problems or solutions of this test during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination at any time via copier, telephone, email, World Wide Web or media of any type is a violation of the competition rules.*

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