Wednesday, FEBRUARY 15, 2006

7th Annual American Mathematics Contest 10

AMC 10

Contest B

The MATHEMATICAL ASSOCIATION OF AMERICA
American Mathematics Competitions

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR GIVES THE SIGNAL TO BEGIN.
2. This is a 25-question, multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Mark your answer to each problem on the AMC 10 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. SCORING: You will receive 6 points for each correct answer, 2.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, ruler, compass, protractor, erasers and calculators that are accepted for use on the SAT. No problems on the test will *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form. When your proctor gives the signal, begin working the problems. You will have **75 MINUTES** to complete the test.
8. When you finish the exam, *sign your name* in the space provided on the Answer Form.

Students who score 120 or above or finish in the top 1% on this AMC 10 will be invited to take the 24th annual American Invitational Mathematics Examination (AIME) on Tuesday, March 7, 2006 or Wednesday, March 22, 2006. More details about the AIME and other information are on the back page of this test booklet.

The Committee on the American Mathematics Competitions (CAMC) reserves the right to re-examine students before deciding whether to grant official status to their scores. The CAMC also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

The publication, reproduction or communication of the problems or solutions of the AMC 10 during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination at any time via copier, telephone, email, World Wide Web or media of any type is a violation of the competition rules.

1. What is $(-1)^1 + (-1)^2 + \cdots + (-1)^{2006}$?

   (A) -2006 **(B)** -1 **(C)** 0 **(D)** 1 **(E)** 2006

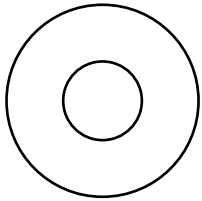
2. For real numbers x and y, define $x \spadesuit y = (x+y)(x-y)$. What is $3 \spadesuit (4 \spadesuit 5)$?

   (A) -72 **(B)** -27 **(C)** -24 **(D)** 24 **(E)** 72

3. A football game was played between two teams, the Cougars and the Panthers. The two teams scored a total of 34 points, and the Cougars won by a margin of 14 points. How many points did the Panthers score?

   (A) 10 **(B)** 14 **(C)** 17 **(D)** 20 **(E)** 24

4. Circles of diameter 1 inch and 3 inches have the same center. The smaller circle is painted red, and the portion outside the smaller circle and inside the larger circle is painted blue. What is the ratio of the blue-painted area to the red-painted area?

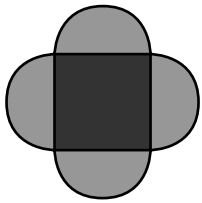


   (A) 2 **(B)** 3 **(C)** 6 **(D)** 8 **(E)** 9

5. A 2×3 rectangle and a 3×4 rectangle are contained within a square without overlapping at any interior point, and the sides of the square are parallel to the sides of the two given rectangles. What is the smallest possible area of the square?

   (A) 16 **(B)** 25 **(C)** 36 **(D)** 49 **(E)** 64

6. A region is bounded by semicircular arcs constructed on the side of a square whose sides measure $2/\pi$, as shown. What is the perimeter of this region?

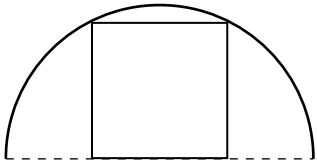


   (A) $\frac{4}{\pi}$ **(B)** 2 **(C)** $\frac{8}{\pi}$ **(D)** 4 **(E)** $\frac{16}{\pi}$

7. Which of the following is equivalent to $\sqrt{\frac{x}{1-\frac{x-1}{x}}}$ when $x<0$?

(A) $-x$ **(B)** x **(C)** 1 **(D)** $\sqrt{\frac{x}{2}}$ **(E)** $x\sqrt{-1}$

8. A square of area 40 is inscribed in a semicircle as shown. What is the area of the semicircle?



(A) 20π **(B)** 25π **(C)** 30π **(D)** 40π **(E)** 50π

9. Francesca uses 100 grams of lemon juice, 100 grams of sugar, and 400 grams of water to make lemonade. There are 25 calories in 100 grams of lemon juice and 386 calories in 100 grams of sugar. Water contains no calories. How many calories are in 200 grams of her lemonade?

(A) 129 **(B)** 137 **(C)** 174 **(D)** 223 **(E)** 411

10. In a triangle with integer side lengths, one side is three times as long as a second side, and the length of the third side is 15. What is the greatest possible perimeter of the triangle?

(A) 43 **(B)** 44 **(C)** 45 **(D)** 46 **(E)** 47

11. What is the tens digit in the sum $7!+8!+9!+\cdots+2006!$?

(A) 1 **(B)** 3 **(C)** 4 **(D)** 6 **(E)** 9

12. The lines $x=\frac{1}{4}y+a$ and $y=\frac{1}{4}x+b$ intersect at the point $(1,2)$. What is $a+b$?

(A) 0 **(B)** $\frac{3}{4}$ **(C)** 1 **(D)** 2 **(E)** $\frac{9}{4}$

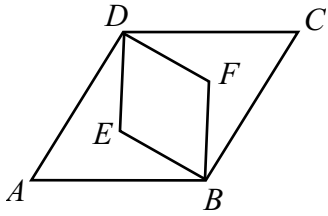
13. Joe and JoAnn each bought 12 ounces of coffee in a 16-ounce cup. Joe drank 2 ounces of his coffee and then added 2 ounces of cream. JoAnn added 2 ounces of cream, stirred the coffee well, and then drank 2 ounces. What is the resulting ratio of the amount of cream in Joe's coffee to that in JoAnn's coffee?

(A) $\frac{6}{7}$ **(B)** $\frac{13}{14}$ **(C)** 1 **(D)** $\frac{14}{13}$ **(E)** $\frac{7}{6}$

14. Let a and b be the roots of the equation $x^2-mx+2=0$. Suppose that $a+(1/b)$ and $b+(1/a)$ are the roots of the equation $x^2-px+q=0$. What is q?

(A) $\frac{5}{2}$ **(B)** $\frac{7}{2}$ **(C)** 4 **(D)** $\frac{9}{2}$ **(E)** 8

15. Rhombus $ABCD$ is similar to rhombus $BFDE$. The area of rhombus $ABCD$ is 24, and $\angle BAD = 60^\circ$. What is the area of rhombus $BFDE$?



(A) 6 **(B)** $4\sqrt{3}$ **(C)** 8 **(D)** 9 **(E)** $6\sqrt{3}$

16. Leap Day, February 29, 2004, occurred on a Sunday. On what day of the week will Leap Day, February 29, 2020, occur?

(A) Tuesday **(B)** Wednesday **(C)** Thursday **(D)** Friday **(E)** Saturday

17. Bob and Alice each have a bag that contains one ball of each of the colors blue, green, orange, red, and violet. Alice randomly selects one ball from her bag and puts it into Bob's bag. Bob then randomly selects one ball from his bag and puts it into Alice's bag. What is the probability that after this process the contents of the two bags are the same?

(A) $\frac{1}{10}$ **(B)** $\frac{1}{6}$ **(C)** $\frac{1}{5}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{1}{2}$

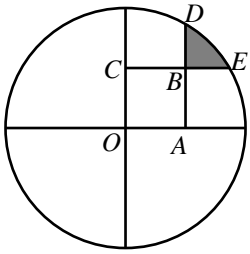
18. Let $a_1, a_2, \ldots$ be a sequence for which

$$a_1 = 2, \quad a_2 = 3, \quad \text{and} \quad a_n = \frac{a_{n-1}}{a_{n-2}} \text{ for each positive integer } n \geq 3.$$

What is a_{2006}?

(A) $\frac{1}{2}$ **(B)** $\frac{2}{3}$ **(C)** $\frac{3}{2}$ **(D)** 2 **(E)** 3

19. A circle of radius 2 is centered at O. Square $OABC$ has side length 1. Sides $\overline{AB}$ and $\overline{CB}$ are extended past B to meet the circle at D and E, respectively. What is the area of the shaded region in the figure, which is bounded by $\overline{BD}$, $\overline{BE}$, and the minor arc connecting D and E?



(A) $\frac{\pi}{3}+1-\sqrt{3}$ **(B)** $\frac{\pi}{2}(2-\sqrt{3})$ **(C)** $\pi(2-\sqrt{3})$ **(D)** $\frac{\pi}{6}+\frac{\sqrt{3}-1}{2}$ **(E)** $\frac{\pi}{3}-1+\sqrt{3}$

20. In rectangle $ABCD$, we have $A=(6,-22)$, $B=(2006,178)$, and $D=(8,y)$, for some integer y. What is the area of rectangle $ABCD$?

(A) 4000 **(B)** 4040 **(C)** 4400 **(D)** 40,000 **(E)** 40,400

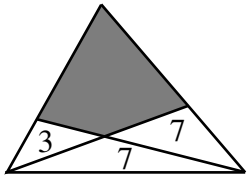
21. For a particular peculiar pair of dice, the probabilities of rolling 1, 2, 3, 4, 5, and 6 on each die are in the ratio $1:2:3:4:5:6$. What is the probability of rolling a total of 7 on the two dice?

(A) $\frac{4}{63}$ **(B)** $\frac{1}{8}$ **(C)** $\frac{8}{63}$ **(D)** $\frac{1}{6}$ **(E)** $\frac{2}{7}$

22. Elmo makes N sandwiches for a fundraiser. For each sandwich he uses B globs of peanut butter at 4¢ per glob and J blobs of jam at 5¢ per blob. The cost of the peanut butter and jam to make all the sandwiches is \$2.53. Assume that B, J, and N are positive integers with $N>1$. What is the cost of the jam Elmo uses to make the sandwiches?

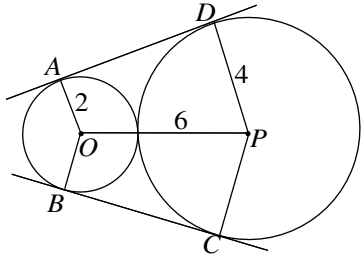
(A) \$1.05 **(B)** \$1.25 **(C)** \$1.45 **(D)** \$1.65 **(E)** \$1.85

23. A triangle is partitioned into three triangles and a quadrilateral by drawing two lines from vertices to their opposite sides. The areas of the three triangles are 3, 7, and 7, as shown. What is the area of the shaded quadrilateral?



(A) 15 **(B)** 17 **(C)** $\frac{35}{2}$ **(D)** 18 **(E)** $\frac{55}{3}$

24. Circles with centers at O and P have radii 2 and 4, respectively, and are externally tangent. Points A and B on the circle with center O and points C and D on the circle with center P are such that $\overline{AD}$ and $\overline{BC}$ are common external tangents to the circles. What is the area of the concave hexagon $AOBCPD$?



(A) $18\sqrt{3}$ **(B)** $24\sqrt{2}$ **(C)** 36 **(D)** $24\sqrt{3}$ **(E)** $32\sqrt{2}$

25. Mr. Jones has eight children of different ages. On a family trip his oldest child, who is 9, spots a license plate with a 4-digit number in which each of two digits appears two times. "Look, daddy!" she exclaims. "That number is evenly divisible by the age of each of us kids!" "That's right," replies Mr. Jones, "and the last two digits just happen to be my age." Which of the following is ***not*** the age of one of Mr. Jones's children?

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

WRITE TO US!

Correspondence about the problems and solutions for this AMC 10 should be addressed to:

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2006 AIME

The AIME will be held on Tuesday, March 7, 2006 with the alternate on Wednesday, March 22, 2006. It is a 15-question, 3-hour, integer-answer exam. You will be invited to participate only if you score 120 or above, or finish in the top 1% of the AMC 10, or if you score 100 or above or finish in the top 5% of the AMC 12. Top-scoring students on the AMC 10/12/AIME will be selected to take the USA Mathematical Olympiad (USAMO) on April 18 and 19, 2006. The best way to prepare for the AIME and USAMO is to study previous exams. Copies may be ordered as indicated below.

PUBLICATIONS

A complete listing of current publications, with ordering instructions, is at our web site:

2006

AMC 10 – Contest B

DO NOT OPEN UNTIL WEDNESDAY, February 15, 2006

****Administration On An Earlier Date Will Disqualify Your School's Results****

1. All information (Rules and Instructions) needed to administer this exam is contained in the TEACHERS' MANUAL, which is outside of this package. **PLEASE READ THE MANUAL BEFORE February 15.** Nothing is needed from inside this package until February 15.
2. Your PRINCIPAL or VICE PRINCIPAL must sign the Certification Form found in the Teachers' Manual.
3. The Answer Forms must be mailed by First Class mail to the AMC no later than 24 hours following the examination.
4. *The publication, reproduction or communication of the problems or solutions of this test during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination at any time via copier, telephone, email, World Wide Web or media of any type is a violation of the competition rules.*

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