

Wednesday, FEBRUARY 21, 2007

8th Annual American Mathematics Contest 10

AMC 10 CONTEST B



THE MATHEMATICAL ASSOCIATION OF AMERICA
American Mathematics Competitions

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR GIVES THE SIGNAL TO BEGIN.
2. This is a 25-question, multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Mark your answer to each problem on the AMC 10 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, ruler, compass, protractor, erasers and calculators that are accepted for use on the SAT. No problems on the test will *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form. When your proctor gives the signal, begin working the problems. You will have 75 MINUTES to complete the test.
8. When you finish the exam, *sign your name* in the space provided on the Answer Form.

Students who score 120 or above or finish in the top 1% on this AMC 10 will be invited to take the 25th annual American Invitational Mathematics Examination (AIME) on Tuesday, March 13, 2007 or Wednesday, March 28, 2007. More details about the AIME and other information are on the back page of this test booklet.

The Committee on the American Mathematics Competitions (CAMC) reserves the right to re-examine students before deciding whether to grant official status to their scores. The CAMC also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

The publication, reproduction or communication of the problems or solutions of the AMC 10 during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via copier, telephone, e-mail, World Wide Web or media of any type during this period is a violation of the competition rules. After the contest period, permission to make copies of individual problems in paper or electronic form including posting on web-pages for educational use is granted without fee provided that copies are not made or distributed for profit or commercial advantage and that copies bear the copyright notice.

1. Isabella's house has 3 bedrooms. Each bedroom is 12 feet long, 10 feet wide, and 8 feet high. Isabella must paint the walls of all the bedrooms. Doorways and windows, which will not be painted, occupy 60 square feet in each bedroom. How many square feet of walls must be painted?

(A) 678 (B) 768 (C) 786 (D) 867 (E) 876

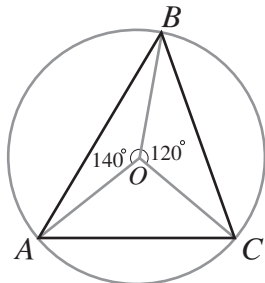
2. Define the operation \star by $a \star b = (a + b)b$. What is $(3 \star 5) - (5 \star 3)$?

(A) -16 (B) -8 (C) 0 (D) 8 (E) 16

3. A college student drove his compact car 120 miles home for the weekend and averaged 30 miles per gallon. On the return trip the student drove his parents' SUV and averaged only 20 miles per gallon. What was the average gas mileage, in miles per gallon, for the round trip?

(A) 22 (B) 24 (C) 25 (D) 26 (E) 28

4. The point O is the center of the circle circumscribed about $\triangle ABC$, with $\angle BOC = 120^\circ$ and $\angle AOB = 140^\circ$, as shown. What is the degree measure of $\angle ABC$?



(A) 35 (B) 40 (C) 45 (D) 50 (E) 60

5. In a certain land, all Arogs are Brafs, all Crups are Brafs, all Dramps are Arogs, and all Crups are Dramps. Which of the following statements is implied by these facts?

(A) All Dramps are Brafs and are Crups.

(B) All Brafs are Crups and are Dramps.

(C) All Arogs are Crups and are Dramps.

(D) All Crups are Arogs and are Brafs.

(E) All Arogs are Dramps and some Arogs may not be Crups.

6. The 2007 AMC 10 will be scored by awarding 6 points for each correct response, 0 points for each incorrect response, and 1.5 points for each problem left unanswered. After looking over the 25 problems, Sarah has decided to attempt the first 22 and leave only the last 3 unanswered. How many of the first 22 problems must she solve correctly in order to score at least 100 points?

(A) 13 (B) 14 (C) 15 (D) 16 (E) 17

7. All sides of the convex pentagon $ABCDE$ are of equal length, and $\angle A = \angle B = 90^\circ$. What is the degree measure of $\angle E$?

(A) 90 (B) 108 (C) 120 (D) 144 (E) 150

8. On the trip home from the meeting where this AMC10 was constructed, the Contest Chair noted that his airport parking receipt had digits of the form $bbcac$, where $0 \leq a < b < c \leq 9$, and b was the average of a and c . How many different five-digit numbers satisfy all these properties?

(A) 12 (B) 16 (C) 18 (D) 20 (E) 24

9. A cryptographic code is designed as follows. The first time a letter appears in a given message it is replaced by the letter that is 1 place to its right in the alphabet (assuming that the letter A is one place to the right of the letter Z). The second time this same letter appears in the given message, it is replaced by the letter that is $1 + 2$ places to the right, the third time it is replaced by the letter that is $1 + 2 + 3$ places to the right, and so on. For example, with this code the word “banana” becomes “cbodqg”. What letter will replace the last letter s in the message

“Lee’s sis is a Mississippi miss, Chriss!”?

(A) g (B) h (C) o (D) s (E) t

10. Two points B and C are in a plane. Let S be the set of all points A in the plane for which $\triangle ABC$ has area 1. Which of the following describes S ?

(A) two parallel lines (B) a parabola (C) a circle (D) a line segment
(E) two points

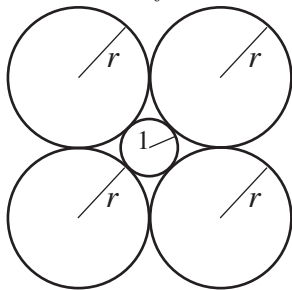
11. A circle passes through the three vertices of an isosceles triangle that has two sides of length 3 and a base of length 2. What is the area of this circle?

(A) 2π (B) $\frac{5}{2}\pi$ (C) $\frac{81}{32}\pi$ (D) 3π (E) $\frac{7}{2}\pi$

12. Tom’s age is T years, which is also the sum of the ages of his three children. His age N years ago was twice the sum of their ages then. What is T/N ?

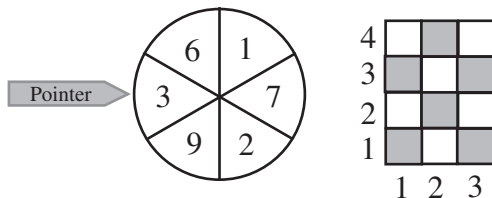
(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

13. Two circles of radius 2 are centered at $(2, 0)$ and at $(0, 2)$. What is the area of the intersection of the interiors of the two circles?
- (A) $\pi - 2$ (B) $\frac{\pi}{2}$ (C) $\frac{\pi\sqrt{3}}{3}$ (D) $2(\pi - 2)$ (E) π
14. Some boys and girls are having a car wash to raise money for a class trip to China. Initially 40% of the group are girls. Shortly thereafter two girls leave and two boys arrive, and then 30% of the group are girls. How many girls were initially in the group?
- (A) 4 (B) 6 (C) 8 (D) 10 (E) 12
15. The angles of quadrilateral $ABCD$ satisfy $\angle A = 2\angle B = 3\angle C = 4\angle D$. What is the degree measure of $\angle A$, rounded to the nearest whole number?
- (A) 125 (B) 144 (C) 153 (D) 173 (E) 180
16. A teacher gave a test to a class in which 10% of the students are juniors and 90% are seniors. The average score on the test was 84. The juniors all received the same score, and the average score of the seniors was 83. What score did each of the juniors receive on the test?
- (A) 85 (B) 88 (C) 93 (D) 94 (E) 98
17. Point P is inside equilateral $\triangle ABC$. Points Q , R , and S are the feet of the perpendiculars from P to \overline{AB} , \overline{BC} , and \overline{CA} , respectively. Given that $PQ = 1$, $PR = 2$, and $PS = 3$, what is AB ?
- (A) 4 (B) $3\sqrt{3}$ (C) 6 (D) $4\sqrt{3}$ (E) 9
18. A circle of radius 1 is surrounded by 4 circles of radius r as shown. What is r ?

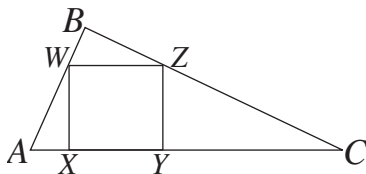


- (A) $\sqrt{2}$ (B) $1 + \sqrt{2}$ (C) $\sqrt{6}$ (D) 3 (E) $2 + \sqrt{2}$

19. The wheel shown is spun twice, and the randomly determined numbers opposite the pointer are recorded. The first number is divided by 4, and the second number is divided by 5. The first remainder designates a column, and the second remainder designates a row on the checkerboard shown. What is the probability that the pair of numbers designates a shaded square?



- (A) $\frac{1}{3}$ (B) $\frac{4}{9}$ (C) $\frac{1}{2}$ (D) $\frac{5}{9}$ (E) $\frac{2}{3}$
20. A set of 25 square blocks is arranged into a 5×5 square. How many different combinations of 3 blocks can be selected from that set so that no two are in the same row or column?
- (A) 100 (B) 125 (C) 600 (D) 2300 (E) 3600
21. Right $\triangle ABC$ has $AB = 3$, $BC = 4$, and $AC = 5$. Square $XYZW$ is inscribed in $\triangle ABC$ with X and Y on \overline{AC} , W on \overline{AB} , and Z on \overline{BC} . What is the side length of the square?



- (A) $\frac{3}{2}$ (B) $\frac{60}{37}$ (C) $\frac{12}{7}$ (D) $\frac{23}{13}$ (E) 2
22. A player chooses one of the numbers 1 through 4. After the choice has been made, two regular four-sided (tetrahedral) dice are rolled, with the sides of the dice numbered 1 through 4. If the number chosen appears on the bottom of exactly one die after it is rolled, then the player wins \$1. If the number chosen appears on the bottom of both of the dice, then the player wins \$2. If the number chosen does not appear on the bottom of either of the dice, the player loses \$1. What is the expected return to the player, in dollars, for one roll of the dice?

- (A) $-\frac{1}{8}$ (B) $-\frac{1}{16}$ (C) 0 (D) $\frac{1}{16}$ (E) $\frac{1}{8}$

23. A pyramid with a square base is cut by a plane that is parallel to its base and is 2 units from the base. The surface area of the smaller pyramid that is cut from the top is half the surface area of the original pyramid. What is the altitude of the original pyramid?

(A) 2 (B) $2 + \sqrt{2}$ (C) $1 + 2\sqrt{2}$ (D) 4 (E) $4 + 2\sqrt{2}$

24. Let n denote the smallest positive integer that is divisible by both 4 and 9, and whose base-10 representation consists of only 4's and 9's, with at least one of each. What are the last four digits of n ?

(A) 4444 (B) 4494 (C) 4944 (D) 9444 (E) 9944

25. How many pairs of positive integers (a, b) are there such that a and b have no common factors greater than 1 and

$$\frac{a}{b} + \frac{14b}{9a}$$

is an integer?

(A) 4 (B) 6 (C) 9 (D) 12 (E) infinitely many

WRITE TO US!

Correspondence about the problems and solutions for this AMC 12 and orders for any of the publications listed below should be addressed to:

American Mathematics Competitions
University of Nebraska, P.O. Box 81606
Lincoln, NE 68501-1606
Phone: 402-472-2257; Fax: 402-472-6087; email: amcinfo@unl.edu

The problems and solutions for this AMC 10 were prepared by the MAA's Committee on the AMC 10 and AMC 12 under the direction of AMC 10 Subcommittee Chair:

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2007 AIME

The AIME will be held on Tuesday, March 13, 2007, with the alternate on Wednesday, March 28, 2007. It is a 15-question, 3-hour, integer-answer exam. You will be invited to participate only if you score 120 or above, or finish in the top 1% of the AMC 10, or if you score 100 or above or finish in the top 5% of the AMC 12. Top-scoring students on the AMC 10/12/AIME will be selected to take the USA Mathematical Olympiad (USAMO) on April 24 and 25, 2007. The best way to prepare for the AIME and USAMO is to study previous exams. Copies may be ordered as indicated below.

PUBLICATIONS

A complete listing of current publications, with ordering instructions, is at our web site:
www.unl.edu/amc.

2007

AMC 10 – CONTEST B

DO NOT OPEN UNTIL

WEDNESDAY, February 21, 2007

****Administration On An Earlier Date Will Disqualify
Your School's Results****

1. All information (Rules and Instructions) needed to administer this exam is contained in the TEACHERS' MANUAL, which is outside of this package. **PLEASE READ THE MANUAL BEFORE February 21.** Nothing is needed from inside this package until February 21.
2. Your PRINCIPAL or VICE PRINCIPAL must sign the Certification Form found in the Teachers' Manual.
3. The Answer Forms must be mailed by First Class mail to the AMC no later than 24 hours following the examination.
4. *The publication, reproduction or communication of the problems or solutions of this test during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination during this period via copier, telephone, email, World Wide Web or media of any type is a violation of the competition rules.*

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