MATHEMATICAL ASSOCIATION OF AMERICA
American Mathematics Competitions

11th Annual

AMC 10 A

American Mathematics Contest 10A

Tuesday, February 9, 2010

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.
2. This is a twenty-five question multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Mark your answer to each problem on the AMC 10 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, rulers, protractors, and erasers. No calculators are allowed. No problems on the test will *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form.
8. When your proctor gives the signal, begin working on the problems. You will have **75 minutes** to complete the test.
9. When you finish the exam, *sign your name* in the space provided on the Answer Form.

The Committee on the American Mathematics Competitions (CAMC) reserves the right to re-examine students before deciding whether to grant official status to their scores. The CAMC also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

Students who score 120 or above or finish in the top 1% on this AMC 10 will be invited to take the 28th annual American Invitational Mathematics Examination (AIME) on Tuesday, March 16, 2010 or Wednesday, March 31, 2010. More details about the AIME and other information are on the back page of this test booklet.

2010

AMC 10

CONTEST A

DO NOT OPEN UNTIL TUESDAY, FEBRUARY 9, 2010

****Administration On An Earlier Date Will Disqualify Your School's Results****

1. All information (Rules and Instructions) needed to administer this exam is contained in the TEACHERS' MANUAL, which is outside of this package. PLEASE READ THE MANUAL BEFORE FEBRUARY 9, 2010. Nothing is needed from inside this package until February 9.
2. Your PRINCIPAL or VICE-PRINCIPAL must verify on the AMC 10 CERTIFICATION FORM (found in the Teachers' Manual) that you followed all rules associated with the conduct of the exam.
3. The Answer Forms must be mailed First Class to the AMC office no later than 24 hours following the exam.
4. *The publication, reproduction or communication of the problems or solutions of this test during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination at any time via copier, telephone, e-mail, World Wide Web or media of any type is a violation of the competition rules.*

The American Mathematics Competitions

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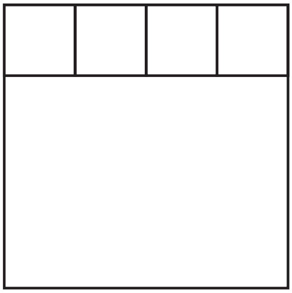
1. Mary's top book shelf holds five books with the following widths, in centimeters: 6, $\frac{1}{2}$, 1, 2.5, and 10. What is the average book width, in centimeters?

   (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

2. Four identical squares and one rectangle are placed together to form one large square as shown. The length of the rectangle is how many times as large as its width?



   (A) $\frac{5}{4}$ **(B)** $\frac{4}{3}$ **(C)** $\frac{3}{2}$ **(D)** 2 **(E)** 3

3. Tyrone had 97 marbles and Eric had 11 marbles. Tyrone then gave some of his marbles to Eric so that Tyrone ended with twice as many marbles as Eric. How many marbles did Tyrone give to Eric?

   (A) 3 **(B)** 13 **(C)** 18 **(D)** 25 **(E)** 29

4. A book that is to be recorded onto compact discs takes 412 minutes to read aloud. Each disc can hold up to 56 minutes of reading. Assume that the smallest possible number of discs is used and that each disc contains the same length of reading. How many minutes of reading will each disc contain?

   (A) 50.2 **(B)** 51.5 **(C)** 52.4 **(D)** 53.8 **(E)** 55.2

5. The area of a circle whose circumference is 24π is $k\pi$. What is the value of k?

   (A) 6 **(B)** 12 **(C)** 24 **(D)** 36 **(E)** 144

6. For positive numbers x and y the operation $\spadesuit(x, y)$ is defined as

$$\spadesuit(x, y) = x - \frac{1}{y}.$$

What is $\spadesuit(2, \spadesuit(2, 2))$?

(A) $\frac{2}{3}$ **(B)** 1 **(C)** $\frac{4}{3}$ **(D)** $\frac{5}{3}$ **(E)** 2

7. Crystal has a running course marked out for her daily run. She starts this run by heading due north for one mile. She then runs northeast for one mile, then southeast for one mile. The last portion of her run takes her on a straight line back to where she started. How far, in miles, is this last portion of her run?

(A) 1 **(B)** $\sqrt{2}$ **(C)** $\sqrt{3}$ **(D)** 2 **(E)** $2\sqrt{2}$

8. Tony works 2 hours a day and is paid \$0.50 per hour for each full year of his age. During a six month period Tony worked 50 days and earned \$630. How old was Tony at the end of the six month period?

(A) 9 **(B)** 11 **(C)** 12 **(D)** 13 **(E)** 14

9. A *palindrome*, such as 83438, is a number that remains the same when its digits are reversed. The numbers x and $x + 32$ are three-digit and four-digit palindromes, respectively. What is the sum of the digits of x?

(A) 20 **(B)** 21 **(C)** 22 **(D)** 23 **(E)** 24

10. Marvin had a birthday on Tuesday, May 27 in the leap year 2008. In what year will his birthday next fall on a Saturday?

(A) 2011 **(B)** 2012 **(C)** 2013 **(D)** 2015 **(E)** 2017

11. The length of the interval of solutions of the inequality $a \le 2x + 3 \le b$ is 10. What is $b - a$?

(A) 6 **(B)** 10 **(C)** 15 **(D)** 20 **(E)** 30

12. Logan is constructing a scaled model of his town. The city's water tower stands 40 meters high, and the top portion is a sphere that holds 100,000 liters of water. Logan's miniature water tower holds 0.1 liters. How tall, in meters, should Logan make his tower?

(A) 0.04 **(B)** $\frac{0.4}{\pi}$ **(C)** 0.4 **(D)** $\frac{4}{\pi}$ **(E)** 4

13. Angelina drove at an average rate of 80 kph and then stopped 20 minutes for gas. After the stop, she drove at an average rate of 100 kph. Altogether she drove 250 km in a total trip time of 3 hours including the stop. Which equation could be used to solve for the time t in hours that she drove before her stop?

(A) $80t + 100(8/3 - t) = 250$ **(B)** $80t = 250$ **(C)** $100t = 250$

(D) $90t = 250$ **(E)** $80(8/3 - t) + 100t = 250$

14. Triangle ABC has $AB = 2 \cdot AC$. Let D and E be on $\overline{AB}$ and $\overline{BC}$, respectively, such that $\angle BAE = \angle ACD$. Let F be the intersection of segments AE and CD, and suppose that $\triangle CFE$ is equilateral. What is $\angle ACB$?

(A) 60° **(B)** 75° **(C)** 90° **(D)** 105° **(E)** 120°

15. In a magical swamp there are two species of talking amphibians: toads, whose statements are always true, and frogs, whose statements are always false. Four amphibians, Brian, Chris, LeRoy, and Mike live together in this swamp, and they make the following statements.

Brian: "Mike and I are different species."
Chris: "LeRoy is a frog."
LeRoy: "Chris is a frog."
Mike: "Of the four of us, at least two are toads."

How many of these four amphibians are frogs?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

16. Nondegenerate $\triangle ABC$ has integer side lengths, $\overline{BD}$ is an angle bisector, $AD = 3$, and $DC = 8$. What is the smallest possible value of the perimeter?

(A) 30 **(B)** 33 **(C)** 35 **(D)** 36 **(E)** 37

17. A solid cube has side length 3 inches. A 2-inch by 2-inch square hole is cut into the center of each face. The edges of each cut are parallel to the edges of the cube, and each hole goes all the way through the cube. What is the volume, in cubic inches, of the remaining solid?

(A) 7 **(B)** 8 **(C)** 10 **(D)** 12 **(E)** 15

18. Bernardo randomly picks 3 distinct numbers from the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and arranges them in descending order to form a 3-digit number. Silvia randomly picks 3 distinct numbers from the set $\{1, 2, 3, 4, 5, 6, 7, 8\}$ and also arranges them in descending order to form a 3-digit number. What is the probability that Bernardo's number is larger than Silvia's number?

(A) $\frac{47}{72}$ **(B)** $\frac{37}{56}$ **(C)** $\frac{2}{3}$ **(D)** $\frac{49}{72}$ **(E)** $\frac{39}{56}$

19. Equiangular hexagon $ABCDEF$ has side lengths $AB = CD = EF = 1$ and $BC = DE = FA = r$. The area of $\triangle ACE$ is 70% of the area of the hexagon. What is the sum of all possible values of r?

(A) $\frac{4\sqrt{3}}{3}$ **(B)** $\frac{10}{3}$ **(C)** 4 **(D)** $\frac{17}{4}$ **(E)** 6

20. A fly trapped inside a cubical box with side length 1 meter decides to relieve its boredom by visiting each corner of the box. It will begin and end in the same corner and visit each of the other corners exactly once. To get from a corner to any other corner, it will either fly or crawl in a straight line. What is the maximum possible length, in meters, of its path?

(A) $4 + 4\sqrt{2}$ **(B)** $2 + 4\sqrt{2} + 2\sqrt{3}$ **(C)** $2 + 3\sqrt{2} + 3\sqrt{3}$ **(D)** $4\sqrt{2} + 4\sqrt{3}$
(E) $3\sqrt{2} + 5\sqrt{3}$

21. The polynomial $x^3 - ax^2 + bx - 2010$ has three positive integer zeros. What is the smallest possible value of a?

(A) 78 **(B)** 88 **(C)** 98 **(D)** 108 **(E)** 118

22. Eight points are chosen on a circle, and chords are drawn connecting every pair of points. No three chords intersect in a single point inside the circle. How many triangles with all three vertices in the interior of the circle are created?

(A) 28 **(B)** 56 **(C)** 70 **(D)** 84 **(E)** 140

23. Each of 2010 boxes in a line contains a single red marble, and for $1 \le k \le 2010$, the box in the k^{th} position also contains k white marbles. Isabella begins at the first box and successively draws a single marble at random from each box, in order. She stops when she first draws a red marble. Let $P(n)$ be the probability that Isabella stops after drawing exactly n marbles. What is the smallest value of n for which $P(n) < \frac{1}{2010}$?

(A) 45 **(B)** 63 **(C)** 64 **(D)** 201 **(E)** 1005

24. The number obtained from the last two nonzero digits of $90!$ is equal to n. What is n?

(A) 12 **(B)** 32 **(C)** 48 **(D)** 52 **(E)** 68

25. Jim starts with a positive integer n and creates a sequence of numbers. Each successive number is obtained by subtracting the largest possible integer square less than or equal to the current number until zero is reached. For example, if Jim starts with $n = 55$, then his sequence contains 5 numbers:

$$\begin{aligned} &55 \\ 55 - 7^2 &= 6 \\ 6 - 2^2 &= 2 \\ 2 - 1^2 &= 1 \\ 1 - 1^2 &= 0 \end{aligned}$$

Let N be the smallest number for which Jim's sequence has 8 numbers. What is the units digit of N?

(A) 1 **(B)** 3 **(C)** 5 **(D)** 7 **(E)** 9

WRITE TO US!

Correspondence about the problems and solutions for this AMC 10 and orders for publications should be addressed to:

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The problems and solutions for this AMC 10 were prepared by the MAA's Committee on the AMC 10 and AMC 12 under the direction of AMC 10 Subcommittee Chair:

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2010 AIME

The 28th annual AIME will be held on Tuesday, March 16, with the alternate on Wednesday, March 31. It is a 15-question, 3-hour, integer-answer exam. You will be invited to participate only if you score 120 or above or finish in the top 1% of the AMC 10, or if you score 100 or above or finish in the top 5% of the AMC 12. Top-scoring students on the AMC 10/12/AIME will be selected to take the 39th Annual USA Mathematical Olympiad (USAMO) on April 27 - 28, 2010. The best way to prepare for the AIME and USAMO is to study previous exams. Copies may be ordered as indicated below.

PUBLICATIONS

A complete listing of current publications, with ordering instructions, is at our web site: www.unl.edu/amc.