


2016
AMC 10A

DO NOT OPEN UNTIL TUESDAY, February 2, 2016

****Administration On An Earlier Date Will Disqualify Your School's Results****

1. All information (Rules and Instructions) needed to administer this exam is contained in the TEACHERS' MANUAL. PLEASE READ THE MANUAL BEFORE FEBRUARY 2, 2016.
2. Your PRINCIPAL or VICE-PRINCIPAL must verify on the AMC 10 CERTIFICATION FORM (found in the Teachers' Manual) that you followed all rules associated with the conduct of the exam.
3. The Answer Forms must be mailed by trackable mail to the AMC office no later than 24 hours following the exam.
4. *The publication, reproduction or communication of the problems or solutions for this contest during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination at any time via copier, telephone, email, internet, or media of any type is a violation of the competition rules.*

The
MAA American Mathematics Competitions
are supported by
The Akamai Foundation
American Mathematical Society
American Statistical Association
Art of Problem Solving
Casualty Actuarial Society
Collaborator's Circle
Conference Board of the Mathematical Sciences
The D.E. Shaw Group
Expil
IDEA MATH
Jane Street Capital
MathWorks
Mu Alpha Theta
National Council of Teachers of Mathematics
Simons Foundation
Society for Industrial and Applied Mathematics
Star League
Tudor Investment Corporation
Two Sigma



MAA100
MATHEMATICAL ASSOCIATION OF AMERICA
CELEBRATING A CENTURY OF ADVANCING MATHEMATICS

American Mathematics Competitions
17th Annual
AMC 10A
American Mathematics Contest 10A
Tuesday, February 2, 2016

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.
2. This is a 25-question multiple-choice test. Each question is followed by answers marked A, B, C, D, and E. Only one of these is correct.
3. Mark your answer to each problem on the AMC 10 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded. **No copies.**
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, rulers, compass, protractors, and erasers. No calculators, smartwatches, or computing devices are allowed. No problems on the test will require the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form.
8. When your proctor gives the signal, begin working on the problems. You will have **75 minutes** to complete the test.
9. When you finish the exam, *sign your name* in the space provided on the Answer Form.

The Committee on the American Mathematics Competitions (CAMC) reserves the right to re-examine students before deciding whether to grant official status to their scores. The CAMC also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

Students who score well on this AMC 10 will be invited to take the 34th annual American Invitational Mathematics Examination (AIME) on Thursday, March 3, 2016 or Wednesday, March 16, 2016. More details about the AIME are on the back page of this test booklet.

The publication, reproduction or communication of the problems or solutions for this contest during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination at any time via copier, telephone, email, internet, or media of any type is a violation of the competition rules.

- What is the value of $\frac{11! - 10!}{9!}$?
(A) 99 (B) 100 (C) 110 (D) 121 (E) 132
- For what value of x does $10^x \cdot 100^{2x} = 1000^5$?
(A) 1 (B) 2 (C) 3 (D) 4 (E) 5
- For every dollar Ben spent on bagels, David spent 25 cents less. Ben paid \$12.50 more than David. How much did they spend in the bagel store together?
(A) \$37.50 (B) \$50.00 (C) \$87.50 (D) \$90.00 (E) \$92.50

- The remainder function can be defined for all real numbers x and y with $y \neq 0$ by

$$\text{rem}(x, y) = x - y \left\lfloor \frac{x}{y} \right\rfloor,$$

where $\left\lfloor \frac{x}{y} \right\rfloor$ denotes the greatest integer less than or equal to $\frac{x}{y}$. What is the value of $\text{rem}\left(\frac{3}{8}, -\frac{2}{5}\right)$?

- (A) $-\frac{3}{8}$ (B) $-\frac{1}{40}$ (C) 0 (D) $\frac{3}{8}$ (E) $\frac{31}{40}$
- A rectangular box has integer side lengths in the ratio 1 : 3 : 4. Which of the following could be the volume of the box?
(A) 48 (B) 56 (C) 64 (D) 96 (E) 144
- Ximena lists the whole numbers 1 through 30 once. Emilio copies Ximena's numbers, replacing each occurrence of the digit 2 by the digit 1. Ximena adds her numbers and Emilio adds his numbers. How much larger is Ximena's sum than Emilio's?
(A) 13 (B) 26 (C) 102 (D) 103 (E) 110
- The mean, median, and mode of the 7 data values 60, 100, x , 40, 50, 200, 90 are all equal to x . What is the value of x ?
(A) 50 (B) 60 (C) 75 (D) 90 (E) 100

- Trickster Rabbit agrees with Foolish Fox to double Fox's money every time Fox crosses the bridge by Rabbit's house, as long as Fox pays 40 coins in toll to Rabbit after each crossing. The payment is made after the doubling. Fox is



American Mathematics Competitions

WRITE TO US!

Correspondence about the problems and solutions for this AMC 10 and orders for publications should be addressed to:

MAA American Mathematics Competitions
PO Box 471

Annapolis Junction, MD 20701

Phone 800.527.3690 | Fax 240.396.5647 | amcinfo@maa.org

The problems and solutions for this AMC 10 were prepared by MAA's Subcommittee on the AMC10/AMC12 Exams, under the direction of the co-chairs Jerrold W. Grossman and Silvia Fernandez.

2016 AIME

The 34th annual AIME will be held on Thursday, March 3, 2016 with the alternate on Wednesday, March 16, 2016. It is a 15-question, 3-hour, integer-answer exam. You will be invited to participate if you achieve a high score on this contest. Top-scoring students on the AMC 10/12/AIME will be selected to take the 45th Annual USA Mathematical Olympiad (USAMO) on April 19–20, 2016. The best way to prepare for the AIME and USAMO is to study previous exams. Copies may be ordered as indicated below.

PUBLICATIONS

A complete listing of current publications, with ordering instructions, is at our web site:
www.maa.org/amc

21. Circles with centers P , Q , and R , having radii 1, 2, and 3, respectively, lie on the same side of line l and are tangent to l at P' , Q' , and R' , respectively, with Q' between P' and R' . The circle with center Q is externally tangent to each of the other two circles. What is the area of $\triangle PQR$?

(A) 0 (B) $\sqrt{\frac{2}{3}}$ (C) 1 (D) $\sqrt{6} - \sqrt{2}$ (E) $\sqrt{\frac{3}{2}}$

22. For some positive integer n , the number $110n^3$ has 110 positive integer divisors, including 1 and the number $110n^3$. How many positive integer divisors does the number $81n^4$ have?

(A) 110 (B) 191 (C) 261 (D) 325 (E) 425

23. A binary operation \diamond has the properties that $a \diamond (b \diamond c) = (a \diamond b) \cdot c$ and that $a \diamond a = 1$ for all nonzero real numbers a , b , and c . (Here the dot \cdot represents the usual multiplication operation.) The solution to the equation $2016 \diamond (6 \diamond x) = 100$ can be written as $\frac{p}{q}$, where p and q are relatively prime positive integers. What is $p + q$?

(A) 109 (B) 201 (C) 301 (D) 3049 (E) 33,601

24. A quadrilateral is inscribed in a circle of radius $200\sqrt{2}$. Three of the sides of this quadrilateral have length 200. What is the length of its fourth side?

(A) 200 (B) $200\sqrt{2}$ (C) $200\sqrt{3}$ (D) $300\sqrt{2}$ (E) 500

25. How many ordered triples (x, y, z) of positive integers satisfy $\text{lcm}(x, y) = 72$, $\text{lcm}(x, z) = 600$, and $\text{lcm}(y, z) = 900$?

(A) 15 (B) 16 (C) 24 (D) 27 (E) 64

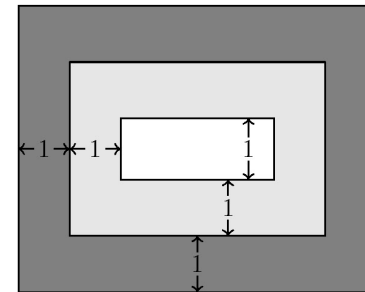
excited about his good fortune until he discovers that all his money is gone after crossing the bridge three times. How many coins did Fox have at the beginning?

(A) 20 (B) 30 (C) 35 (D) 40 (E) 45

9. A triangular array of 2016 coins has 1 coin in the first row, 2 coins in the second row, 3 coins in the third row, and so on up to N coins in the N th row. What is the sum of the digits of N ?

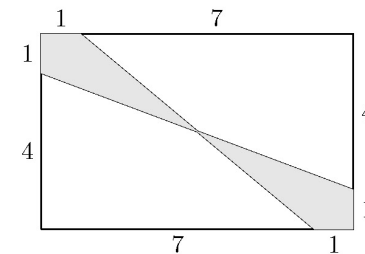
(A) 6 (B) 7 (C) 8 (D) 9 (E) 10

10. A rug is made with three different colors as shown. The areas of the three differently colored regions form an arithmetic progression. The inner rectangle is one foot wide, and each of the two shaded regions is 1 foot wide on all four sides. What is the length in feet of the inner rectangle?



(A) 1 (B) 2 (C) 4 (D) 6 (E) 8

11. What is the area of the shaded region of the given 8×5 rectangle?



(A) $4\frac{3}{4}$ (B) 5 (C) $5\frac{1}{4}$ (D) $6\frac{1}{2}$ (E) 8

12. Three distinct integers are selected at random between 1 and 2016, inclusive. Which of the following is a correct statement about the probability p that the product of the three integers is odd?

(A) $p < \frac{1}{8}$ (B) $p = \frac{1}{8}$ (C) $\frac{1}{8} < p < \frac{1}{3}$ (D) $p = \frac{1}{3}$ (E) $p > \frac{1}{3}$

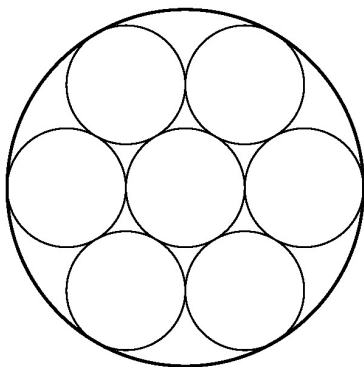
13. Five friends sat in a movie theater in a row containing 5 seats, numbered 1 to 5 from left to right. (The directions “left” and “right” are from the point of view of the people as they sit in the seats.) During the movie Ada went to the lobby to get some popcorn. When she returned, she found that Bea had moved two seats to the right, Ceci had moved one seat to the left, and Dee and Edie had switched seats, leaving an end seat for Ada. In which seat had Ada been sitting before she got up?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

14. How many ways are there to write 2016 as the sum of twos and threes, ignoring order? (For example, $1008 \cdot 2 + 0 \cdot 3$ and $402 \cdot 2 + 404 \cdot 3$ are two such ways.)

(A) 236 (B) 336 (C) 337 (D) 403 (E) 672

15. Seven cookies of radius 1 inch are cut from a circle of cookie dough, as shown. Neighboring cookies are tangent, and all except the center cookie are tangent to the edge of the dough. The leftover scrap is reshaped to form another cookie of the same thickness. What is the radius in inches of the scrap cookie?



(A) $\sqrt{2}$ (B) 1.5 (C) $\sqrt{\pi}$ (D) $\sqrt{2\pi}$ (E) π

16. A triangle with vertices $A(0, 2)$, $B(-3, 2)$, and $C(-3, 0)$ is reflected about the x -axis; then the image $\triangle A'B'C'$ is rotated counterclockwise around the origin by 90° to produce $\triangle A''B''C''$. Which of the following transformations will return $\triangle A''B''C''$ to $\triangle ABC$?

(A) counterclockwise rotation around the origin by 90°

(B) clockwise rotation around the origin by 90°

(C) reflection about the x -axis

(D) reflection about the line $y = x$

(E) reflection about the y -axis

17. Let N be a positive multiple of 5. One red ball and N green balls are arranged in a line in random order. Let $P(N)$ be the probability that at least $\frac{3}{5}$ of the green balls are on the same side of the red ball. Observe that $P(5) = 1$ and that $P(N)$ approaches $\frac{4}{5}$ as N grows large. What is the sum of the digits of the least value of N such that $P(N) < \frac{321}{400}$?

(A) 12 (B) 14 (C) 16 (D) 18 (E) 20

18. Each vertex of a cube is to be labeled with an integer from 1 through 8, with each integer being used once, in such a way that the sum of the four numbers on the vertices of a face is the same for each face. Arrangements that can be obtained from each other through rotations of the cube are considered to be the same. How many different arrangements are possible?

(A) 1 (B) 3 (C) 6 (D) 12 (E) 24

19. In rectangle $ABCD$, $AB = 6$ and $BC = 3$. Point E between B and C , and point F between E and C are such that $BE = EF = FC$. Segments \overline{AE} and \overline{AF} intersect \overline{BD} at P and Q , respectively. The ratio $BP : PQ : QD$ can be written as $r : s : t$, where the greatest common factor of r , s , and t is 1. What is $r + s + t$?

(A) 7 (B) 9 (C) 12 (D) 15 (E) 20

20. For some particular value of N , when $(a + b + c + d + 1)^N$ is expanded and like terms are combined, the resulting expression contains exactly 1001 terms that include all four variables, a , b , c , and d , each to some positive power. What is N ?

(A) 9 (B) 14 (C) 16 (D) 17 (E) 19