



MAA AMC
American Mathematics Competitions

American Mathematics Competitions

19th Annual

AMC 10B

American Mathematics Competition 10B

Thursday, February 15, 2018

INSTRUCTIONS

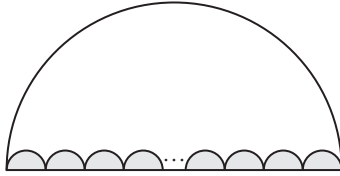
1. DO NOT OPEN THIS BOOKLET UNTIL YOUR COMPETITION MANAGER TELLS YOU.
2. This is a 25-question multiple-choice exam. Each question is followed by answers marked A, B, C, D, and E. Only one of these is correct.
3. Mark your answer to each problem on the AMC 10 Answer Sheet with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer sheet will be graded. **You must use and submit the original answer sheets provided by the MAA AMC. Photocopies will not be scored.**
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. Only scratch paper, graph paper, rulers, compasses, protractors, and erasers are allowed as aids. No calculators, smartwatches, phones, or computing devices are allowed. No problems on the exam will require the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the exam, your competition manager will ask you to record certain information on the answer form.
8. When your competition manager gives the signal, begin working on the problems. You will have **75 minutes** to complete the exam.
9. When you finish the exam, *sign your name* in the space provided at the top of the Answer Sheet.

The Committee on the American Mathematics Competitions reserves the right to disqualify scores from a school if it determines that the required security procedures were not followed.

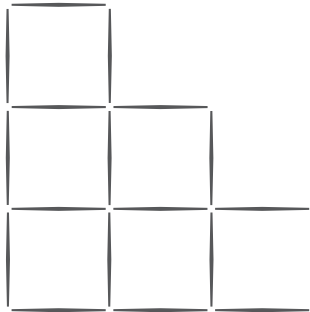
Students who score well on this AMC 10 will be invited to take the 36th annual American Invitational Mathematics Examination (AIME) on Tuesday, March 6, 2018 or Wednesday, March 21, 2018. More details about the AIME are on the back page of this exam booklet.

- Kate bakes a 20-inch by 18-inch pan of cornbread. The cornbread is cut into pieces that measure 2 inches by 2 inches. How many pieces of cornbread does the pan contain?
(A) 90 (B) 100 (C) 180 (D) 200 (E) 360
- Sam drove 96 miles in 90 minutes. His average speed during the first 30 minutes was 60 mph (miles per hour), and his average speed during the second 30 minutes was 65 mph. What was his average speed, in mph, during the last 30 minutes?
(A) 64 (B) 65 (C) 66 (D) 67 (E) 68
- In the expression $(__ \times __) + (__ \times __)$ each blank is to be filled in with one of the digits 1, 2, 3, or 4, with each digit being used once. How many different values can be obtained?
(A) 2 (B) 3 (C) 4 (D) 6 (E) 24
- A three-dimensional rectangular box with dimensions X , Y , and Z has faces whose surface areas are 24, 24, 48, 48, 72, and 72 square units. What is $X + Y + Z$?
(A) 18 (B) 22 (C) 24 (D) 30 (E) 36
- How many subsets of $\{2, 3, 4, 5, 6, 7, 8, 9\}$ contain at least one prime number?
(A) 128 (B) 192 (C) 224 (D) 240 (E) 256
- A box contains 5 chips, numbered 1, 2, 3, 4, and 5. Chips are drawn randomly one at a time without replacement until the sum of the values drawn exceeds 4. What is the probability that 3 draws are required?
(A) $\frac{1}{15}$ (B) $\frac{1}{10}$ (C) $\frac{1}{6}$ (D) $\frac{1}{5}$ (E) $\frac{1}{4}$

7. In the figure below, N congruent semicircles are drawn along a diameter of a large semicircle, with their diameters covering the diameter of the large semicircle with no overlap. Let A be the combined area of the small semicircles and B be the area of the region inside the large semicircle but outside the small semicircles. The ratio $A : B$ is $1 : 18$. What is N ?

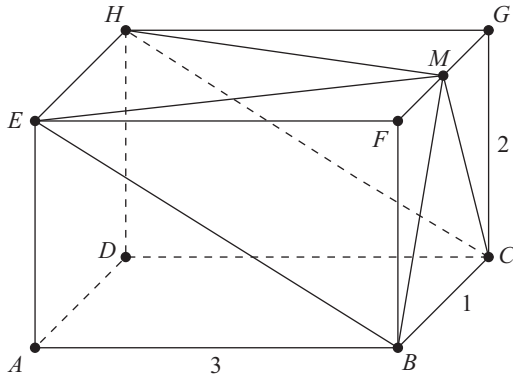


- (A) 16 (B) 17 (C) 18 (D) 19 (E) 36
8. Sara makes a staircase out of toothpicks as shown:



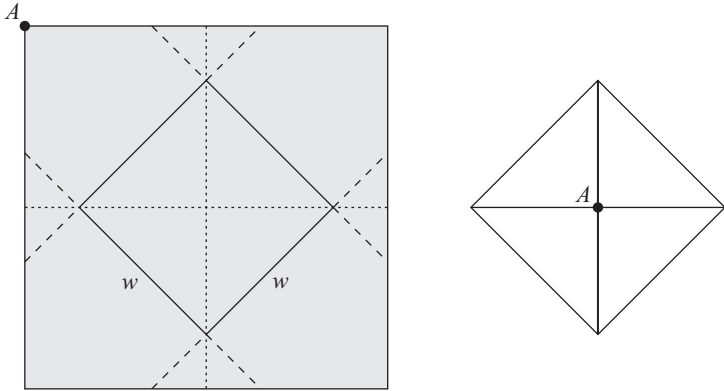
- This is a 3-step staircase and uses 18 toothpicks. How many steps would be in a staircase that used 180 toothpicks?
- (A) 10 (B) 11 (C) 12 (D) 24 (E) 30
9. The faces of each of 7 standard dice are labeled with the integers from 1 to 6. Let p be the probability that when all 7 dice are rolled, the sum of the numbers on the top faces is 10. What other sum occurs with the same probability p ?
- (A) 13 (B) 26 (C) 32 (D) 39 (E) 42

10. In the rectangular parallelepiped shown, $AB = 3$, $BC = 1$, and $CG = 2$. Point M is the midpoint of \overline{FG} . What is the volume of the rectangular pyramid with base $BCHE$ and apex M ?



- (A) 1 (B) $\frac{4}{3}$ (C) $\frac{3}{2}$ (D) $\frac{5}{3}$ (E) 2
11. Which of the following expressions is never a prime number when p is a prime number?
- (A) $p^2 + 16$ (B) $p^2 + 24$ (C) $p^2 + 26$ (D) $p^2 + 46$
 (E) $p^2 + 96$
12. Line segment \overline{AB} is a diameter of a circle with $AB = 24$. Point C , not equal to A or B , lies on the circle. As point C moves around the circle, the centroid (center of mass) of $\triangle ABC$ traces out a closed curve missing two points. To the nearest positive integer, what is the area of the region bounded by this curve?
- (A) 25 (B) 38 (C) 50 (D) 63 (E) 75
13. How many of the first 2018 numbers in the sequence 101, 1001, 10001, 100001, ... are divisible by 101?
- (A) 253 (B) 504 (C) 505 (D) 506 (E) 1009
14. A list of 2018 positive integers has a unique mode, which occurs exactly 10 times. What is the least number of distinct values that can occur in the list?
- (A) 202 (B) 223 (C) 224 (D) 225 (E) 234

15. A closed box with a square base is to be wrapped with a square sheet of wrapping paper. The box is centered on the wrapping paper with the vertices of the base lying on the midlines of the square sheet of paper, as shown in the figure on the left. The four corners of the wrapping paper are to be folded up over the sides and brought together to meet at the center of the top of the box, point A in the figure on the right. The box has base length w and height h . What is the area of the sheet of wrapping paper?



- (A) $2(w + h)^2$ (B) $\frac{(w + h)^2}{2}$ (C) $2w^2 + 4wh$ (D) $2w^2$
 (E) w^2h
16. Let $a_1, a_2, \dots, a_{2018}$ be a strictly increasing sequence of positive integers such that

$$a_1 + a_2 + \dots + a_{2018} = 2018^{2018}.$$

What is the remainder when $a_1^3 + a_2^3 + \dots + a_{2018}^3$ is divided by 6?

- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4
17. In rectangle $PQRS$, $PQ = 8$ and $QR = 6$. Points A and B lie on \overline{PQ} , points C and D lie on \overline{QR} , points E and F lie on \overline{RS} , and points G and H lie on \overline{SP} so that $AP = BQ < 4$ and the convex octagon $ABCDEFGH$ is equilateral. The length of a side of this octagon can be expressed in the form $k + m\sqrt{n}$, where k , m , and n are integers and n is not divisible by the square of any prime. What is $k + m + n$?
- (A) 1 (B) 7 (C) 21 (D) 92 (E) 106

18. Three young brother-sister pairs from different families need to take a trip in a van. These six children will occupy the second and third rows in the van, each of which has three seats. To avoid disruptions, siblings may not sit right next to each other in the same row, and no child may sit directly in front of his or her sibling. How many seating arrangements are possible for this trip?

(A) 60 (B) 72 (C) 92 (D) 96 (E) 120

19. Joey and Chloe and their daughter Zoe all have the same birthday. Joey is 1 year older than Chloe, and Zoe is exactly 1 year old today. Today is the first of the 9 birthdays on which Chloe's age will be an integral multiple of Zoe's age. What will be the sum of the two digits of Joey's age the next time his age is a multiple of Zoe's age?

(A) 7 (B) 8 (C) 9 (D) 10 (E) 11

20. A function f is defined recursively by $f(1) = f(2) = 1$ and

$$f(n) = f(n-1) - f(n-2) + n$$

for all integers $n \geq 3$. What is $f(2018)$?

(A) 2016 (B) 2017 (C) 2018 (D) 2019 (E) 2020

21. Mary chose an even 4-digit number n . She wrote down all the divisors of n in increasing order from left to right: $1, 2, \dots, \frac{n}{2}, n$. At some moment Mary wrote 323 as a divisor of n . What is the smallest possible value of the next divisor written to the right of 323?

(A) 324 (B) 330 (C) 340 (D) 361 (E) 646

22. Real numbers x and y are chosen independently and uniformly at random from the interval $[0, 1]$. Which of the following numbers is closest to the probability that x , y , and 1 are the side lengths of an obtuse triangle?

(A) 0.21 (B) 0.25 (C) 0.29 (D) 0.50 (E) 0.79

23. How many ordered pairs (a, b) of positive integers satisfy the equation

$$a \cdot b + 63 = 20 \cdot \text{lcm}(a, b) + 12 \cdot \text{gcd}(a, b),$$

where $\text{gcd}(a, b)$ denotes the greatest common divisor of a and b , and $\text{lcm}(a, b)$ denotes their least common multiple?

(A) 0 (B) 2 (C) 4 (D) 6 (E) 8

24. Let $ABCDEF$ be a regular hexagon with side length 1. Denote by X , Y , and Z the midpoints of sides \overline{AB} , \overline{CD} , and \overline{EF} , respectively. What is the area of the convex hexagon whose interior is the intersection of the interiors of $\triangle ACE$ and $\triangle XYZ$?

(A) $\frac{3}{8}\sqrt{3}$ (B) $\frac{7}{16}\sqrt{3}$ (C) $\frac{15}{32}\sqrt{3}$ (D) $\frac{1}{2}\sqrt{3}$ (E) $\frac{9}{16}\sqrt{3}$

25. Let $\lfloor x \rfloor$ denote the greatest integer less than or equal to x . How many real numbers x satisfy the equation $x^2 + 10,000\lfloor x \rfloor = 10,000x$?

(A) 197 (B) 198 (C) 199 (D) 200 (E) 201



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Questions and comments about problems
and solutions for this exam should be sent to:

amcq@maa.org

Send questions and comments about administrative arrangements to:

amcinfo@maa.org

or

MAA American Mathematics Competitions
P.O. Box 471
Annapolis Junction, MD 20701

*The problems and solutions for this AMC 10 were prepared by MAA's
Subcommittee on the AMC10/AMC12 Exams, under the direction of the co-chairs
Jerrold W. Grossman and Carl Yerger.*

2018 AIME

The 36th annual AIME will be held on Tuesday, March 6, 2018 with the alternate on Wednesday, March 21, 2018. It is a 15-question, 3-hour, integer-answer exam. You will be invited to participate if you achieve a high score on this competition. Top-scoring students on the AMC 10/12/AIME will be selected to take the 47th Annual USA Mathematical Olympiad (USAMO) on April 18–19, 2018.

2018

AMC 10B



DO NOT OPEN UNTIL THURSDAY, February 15, 2018

****Administration On An Earlier Date Will Disqualify Your School's Results****

1. All the information needed to administer this exam is contained in the AMC 10/12 Teacher's Manual. PLEASE READ THE MANUAL BEFORE FEBRUARY 15, 2018.
 2. Your PRINCIPAL or VICE PRINCIPAL must verify on the AMC 10/12 COMPETITION CERTIFICATION FORM (found on maa.org/amc under 'AMC 10/12') that you followed all rules associated with the administration of the exam.
 3. Answer sheets must be returned to the MAA AMC office the day after the competition. Ship with appropriate postage using a trackable method. FedEx or UPS is strongly recommended.
 4. The publication, reproduction, or communication of the problems or solutions of this exam during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via phone, email, or digital media of any type during this period is a violation of the competition rules.
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