MAA
MATHEMATICAL ASSOCIATION OF AMERICA

Solutions Pamphlet
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AMC 10B

American Mathematics Competition 10B
Thursday, February 15, 2018

This Pamphlet gives at least one solution for each problem on this year's competition and shows that all problems can be solved without the use of a calculator. When more than one solution is provided, this is done to illustrate a significant contrast in methods, e.g., algebraic versus geometric, computational versus conceptual, elementary versus advanced. These solutions are by no means the only ones possible, nor are they superior to others the reader may devise.

We hope that teachers will inform their students about these solutions, both as illustrations of the kinds of ingenuity needed to solve nonroutine problems and as examples of good mathematical exposition. *However, the publication, reproduction, or communication of the problems or solutions for this contest during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination at any time via copier, telephone, email, internet, or media of any type is a violation of the competition rules.*

1. **Answer (A):** The total area of cornbread is $20 \cdot 18 = 360$ in^2. Because each piece of cornbread has area $2 \cdot 2 = 4$ in^2, the pan contains $360 \div 4 = 90$ pieces of cornbread.

<div align="center">

OR

</div>

When cut, there are $20 \div 2 = 10$ pieces of cornbread along a long side of the pan and $18 \div 2 = 9$ pieces along a short side, so there are $10 \cdot 9 = 90$ pieces.

2. **Answer (D):** Sam covered $\frac{1}{2} \cdot 60 = 30$ miles during the first 30 minutes and $\frac{1}{2} \cdot 65 = 32.5$ miles during the second 30 minutes, so he needed to cover $96 - 30 - 32.5 = 33.5$ miles during the last 30 minutes. Thus his average speed during the last 30 minutes was

$$\frac{33.5 \text{ miles}}{\frac{1}{2} \text{ hour}} = 67 \text{ mph}.$$

3. **Answer (B):** Both the multiplications and the addition can be performed in either order, so each possible value can be obtained by putting the 1 in the first position and one of the other three numbers in the second position. Therefore the only possible values are

$$(1 \times 2) + (3 \times 4) = 14,$$
$$(1 \times 3) + (2 \times 4) = 11,$$
$$\text{and} \quad (1 \times 4) + (2 \times 3) = 10,$$

so just 3 different values can be obtained.

4. **Answer (B):** Without loss of generality, assume that $X \leq Y \leq Z$. Then the geometric description of the problem can be translated into the system of equations, $XY = 24$, $XZ = 48$, and $YZ = 72$. Dividing the second equation by the first yields $\frac{Z}{Y} = 2$, so $Z = 2Y$. Then $72 = YZ = 2Y^2$, so $Y^2 = 36$. Because Y is positive, $Y = 6$. It follows that $X = 24 \div 6 = 4$ and $Z = 72 \div 6 = 12$, so $X + Y + Z = 22$.

<div align="center">

OR

</div>

With X, Y, and Z as above, multiply the three equations to give

$$X^2Y^2Z^2 = 24 \cdot 48 \cdot 72 = 24 \cdot 24 \cdot 2 \cdot 24 \cdot 3 = 24^2 \cdot 144 = (24 \cdot 12)^2.$$

Therefore $XYZ = 24 \cdot 12$, and dividing successively by the three equations gives $Z = 12$, $Y = 6$, and $X = 4$, so $X + Y + Z = 22$.

5. **Answer (D):** The number of qualifying subsets equals the difference between the total number of subsets of $\{2, 3, 4, 5, 6, 7, 8, 9\}$ and the number of such subsets containing no prime numbers, which is the number of subsets of $\{4, 6, 8, 9\}$. A set with n elements has 2^n subsets, so the requested number is $2^8 - 2^4 = 256 - 16 = 240$.

<div align="center">**OR**</div>

A subset meeting the condition must be the union of a nonempty subset of $\{2, 3, 5, 7\}$ and a subset of $\{4, 6, 8, 9\}$. There are $2^4 - 1 = 15$ of the former and $2^4 = 16$ of the latter, which gives $15 \cdot 16 = 240$ choices in all.

6. **Answer (D):** Three draws will be required if and only if the first two chips drawn have a sum of 4 or less. The draws $(1, 2)$, $(2, 1)$, $(1, 3)$, and $(3, 1)$ are the only draws meeting this condition. There are $5 \cdot 4 = 20$ possible two-chip draws, so the requested probability is $\frac{4}{20} = \frac{1}{5}$. (Note that all 20 possible two-chip draws are considered in determining the denominator, even though some draws will end after the first chip is drawn.)

7. **Answer (D):** Suppose without loss of generality that each small semicircle has radius 1; then the large semicircle has radius N. The area of each small semicircle is $\frac{\pi}{2}$, and the area of the large semicircle is $N^2 \cdot \frac{\pi}{2}$. The combined area A of the N small semicircles is $N \cdot \frac{\pi}{2}$, and the area B inside the large semicircle but outside the small semicircles is

$$N^2 \cdot \frac{\pi}{2} - N \cdot \frac{\pi}{2} = \left(N^2 - N\right) \cdot \frac{\pi}{2}.$$

Thus the ratio $A : B$ of the areas is $N : \left(N^2 - N\right)$, which is $1 : (N-1)$. Because this ratio is given to be $1 : 18$, it follows that $N - 1 = 18$ and $N = 19$.

8. **Answer (C):** In the staircase with n steps, the number of vertical toothpicks is

$$1 + 2 + 3 + \cdots + n + n = \frac{n(n+1)}{2} + n.$$

There are an equal number of horizontal toothpicks, for a total of $n(n+1) + 2n$ toothpicks. Solving $n(n+1) + 2n = 180$ with $n > 0$ yields $n = 12$.

OR

By inspection, the number of toothpicks for staircases consisting of 1, 2, and 3 steps are 4, 10, and 18, respectively. The n-step staircase is obtained from the $(n-1)$-step staircase by adding $n+1$ horizontal toothpicks and $n+1$ vertical toothpicks. With this observation, the pattern can be continued so that 28, 40, 54, 70, 88, 108, 130, 154, and 180 are the numbers of toothpicks used to construct staircases consisting of 4 through 12 steps, respectively. Therefore 180 toothpicks are needed for the 12-step staircase.

9. **Answer (D):** Without loss of generality, one can assume that the numbers on opposite faces of each die add up to 7. In other words, the 1 is opposite the 6, the 2 is opposite the 5, and the 3 is opposite the 4. (In fact, standard dice are numbered in this way.) The top faces give a sum of 10 if and only if the bottom faces give a sum of $7 \cdot 7 - 10 = 39$. By symmetry, the probability that the top faces give a sum of 39 is also p. The distribution of the outcomes of the dice rolls has the bell-shaped graph shown below, so no other outcome has the same probability as 10 and 39.
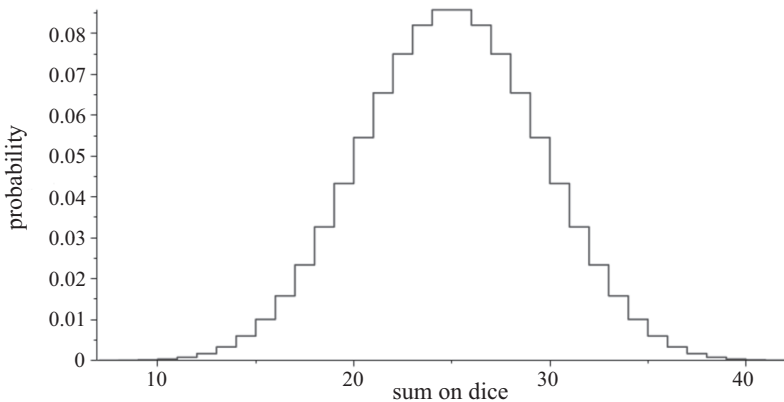
10. **Answer (E):** The volume of the rectangular pyramid with base $BCHE$ and apex M equals the volume of the given rectangular parallelepiped, which is 6, minus the combined volume of triangular prism $AEHDCB$, tetrahedron $BEFM$, and tetrahedron $CGHM$. Tetrahedra $BEFM$ and $CGHM$ each have three right angles at F and G, respectively, and the edges of the tetrahedra emanating from F and G have lengths 2, 3, and $\frac{1}{2}$, so the volume of each of these tetrahedra

is $\frac{1}{6} \cdot (2 \cdot 3 \cdot \frac{1}{2}) = \frac{1}{2}$. The volume of the triangluar prism $AEHDCB$ is 3 because it is half the volume of the rectangular parallelepiped. Therefore the requested volume is $6 - 3 - \frac{1}{2} - \frac{1}{2} = 2$.
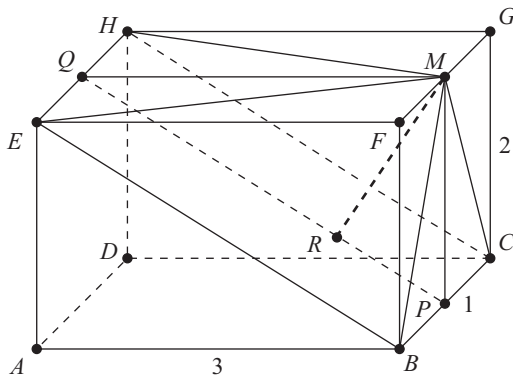
OR

Let P and Q be the midpoints of $\overline{BC}$ and $\overline{EH}$, respectively. By the Pythagorean Theorem $PQ = \sqrt{13}$. Let R be the foot of the perpendicular from M to $\overline{PQ}$. Then $\triangle PMQ \sim \triangle PRM$, so

$$\frac{3}{\sqrt{13}} = \frac{MQ}{PQ} = \frac{RM}{PM} = \frac{RM}{2} \qquad \text{and} \qquad RM = \frac{6}{\sqrt{13}}.$$

The requested volume of the pyramid is $\frac{1}{3}$ times the area of the base times the height, which is

$$\frac{1}{3} \cdot \left(\sqrt{13} \cdot 1 \right) \cdot \frac{6}{\sqrt{13}} = 2.$$

11. **Answer (C):** If $p = 3$, then $p^2 + 26 = 35 = 5 \cdot 7$. If p is a prime number other than 3, then $p = 3k \pm 1$ for some positive integer k. In that case

$$p^2 + 26 = (3k \pm 1)^2 + 26 = 9k^2 \pm 6k + 27 = 3(3k^2 \pm 2k + 9)$$

is a multiple of 3 and is not prime. The smallest counterexamples for the other choices are $5^2 + 16 = 41$, $7^2 + 24 = 73$, $5^2 + 46 = 71$, and $19^2 + 96 = 457$.

12. **Answer (C):** Let O be the center of the circle. Triangle ABC is a right triangle, and O is the midpoint of the hypotenuse $\overline{AB}$. Then

$\overline{OC}$ is a radius, and it is also one of the medians of the triangle. The centroid is located one third of the way along the median from O to C, so the centroid traces out a circle with center O and radius $\frac{1}{3} \cdot 12 = 4$ (except for the two missing points corresponding to $C = A$ or $C = B$). The area of this smaller circle is then $\pi \cdot 4^2 = 16\pi \approx 16 \cdot 3.14 \approx 50$.

13. **Answer (C):**   The numbers in the given sequence are of the form $10^n + 1$ for $n = 2, 3, \ldots, 2019$. If n is even, say $n = 2k$ for some positive integer k, then $10^n + 1 = 100^k + 1 \equiv (-1)^k + 1 \pmod{101}$. Thus $10^n + 1$ is divisible by 101 if and only if k is odd, which means $n = 2, 6, 10, \ldots, 2018$. There are $\frac{1}{4}(2018 - 2) + 1 = 505$ such values. On the other hand, if n is odd, say $n = 2k + 1$ for some positive integer k, then

$$10^n + 1 = 10 \cdot 10^{n-1} + 1 = 10 \cdot 100^k + 1 \equiv 10 \cdot (-1)^k + 1 \pmod{101},$$

which is congruent to 9 or 11, and $10^n + 1$ is not divisible by 101 in this case.

14. **Answer (D):**   The list has $2018 - 10 = 2008$ entries that are not equal to the mode. Because the mode is unique, each of these 2008 entries can occur at most 9 times. There must be at least $\left\lceil \frac{2008}{9} \right\rceil = 224$ distinct values in the list that are different from the mode, because if there were fewer than this many such values, then the size of the list would be at most $9 \cdot \left(\left\lceil \frac{2008}{9} \right\rceil - 1 \right) + 10 = 2017 < 2018$. (The ceiling function notation $\lceil x \rceil$ represents the least integer greater than or equal to x.) Therefore the least possible number of distinct values that can occur in the list is 225. One list satisfying the conditions of the problem contains 9 instances of each of the numbers 1 through 223, 10 instances of the number 224, and one instance of 225.

15. **Answer (A):**   The figure shows that the distance AO from a corner of the wrapping paper to the center is

$$\frac{w}{2} + h + \frac{w}{2} = w + h.$$

The side of the wrapping paper, $\overline{AB}$ in the figure, is the hypotenuse of a $45 - 45 - 90°$ right triangle, so its length is $\sqrt{2} \cdot AO = \sqrt{2}(w + h)$. Therefore the area of the wrapping paper is

$$\left(\sqrt{2}(w + h) \right)^2 = 2(w + h)^2.$$
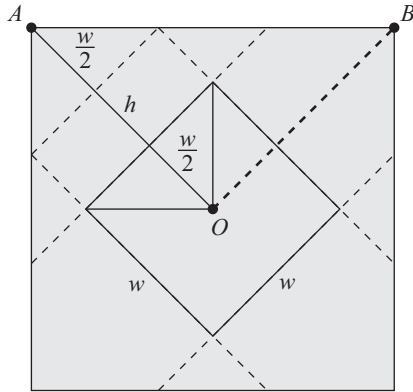
OR

The area of the wrapping paper, excluding the four small triangles indicated by the dashed lines, is equal to the surface area of the box, which is $2w^2 + 4wh$. The four triangles are isosceles right triangles with leg length h, so their combined area is $4 \cdot \frac{1}{2}h^2 = 2h^2$. Thus the total area of the wrapping paper is $2w^2 + 4wh + 2h^2 = 2(w+h)^2$.

16. **Answer (E):** Let n be an integer. Because $n^3 - n = (n-1)n(n+1)$, it follows that $n^3 - n$ has at least one prime factor of 2 and one prime factor of 3 and therefore is divisible by 6. Thus $n^3 \equiv n \pmod{6}$. Then

$$a_1^3 + a_2^3 + \cdots + a_{2018}^3 \equiv a_1 + a_2 + \cdots + a_{2018} \equiv 2018^{2018} \pmod{6}.$$

Because $2018 \equiv 2 \pmod{6}$, the powers of 2018 modulo 6 are alternately $2, 4, 2, 4, \ldots$, so $2018^{2018} \equiv 4 \pmod{6}$. Therefore the remainder when $a_1^3 + a_2^3 + \cdots + a_{2018}^3$ is divided by 6 is 4.

17. **Answer (B):** Because $AP < 4 = \frac{1}{2}PQ$, it follows that A is closer to P than it is to Q and that A is between points P and B. Because $AP = BQ$, $AH = BC$, and angles APH and BQC are right angles, $\triangle APH \cong \triangle BQC$. Thus $PH = QC$, and $PQCH$ is a rectangle. Because $CD = HG$, it follows that $HCDG$ is also a rectangle. Thus $GDRS$ is a rectangle and $DR = GS$, and it follows that $\triangle ERD \cong \triangle FSG$. Therefore segment $\overline{EF}$ is centered in $\overline{RS}$ just as congruent segment $\overline{AB}$ is centered in $\overline{PQ}$. Therefore $\triangle ERD \cong \triangle BQC$, and $\overline{CD}$ is also centered in $\overline{QR}$. Let $2x$ be the side length $AB = BC =$

$CD = DE = EF = FG = GH = HA$ of the regular octagon; then $AP = BQ = 4-x$ and $QC = RD = 3-x$. Applying the Pythagorean Theorem to $\triangle BQC$ yields $(4-x)^2+(3-x)^2 = (2x)^2$, which simplifies to $2x^2 + 14x - 25 = 0$. Thus $x = \frac{1}{2} \cdot (-7 \pm 3\sqrt{11})$, and because $x > 0$, it follows that $2x = -7 + 3\sqrt{11}$. Hence $k + m + n = -7 + 3 + 11 = 7$.
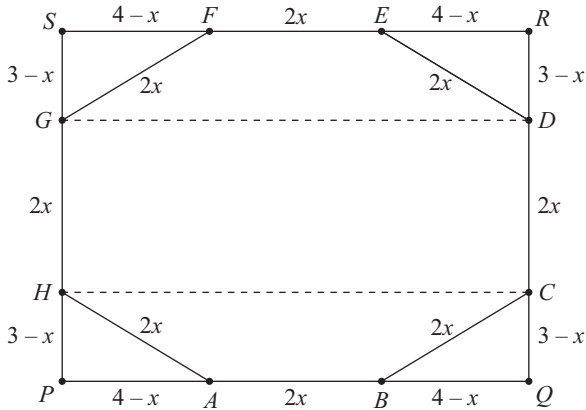
18. **Answer (D):** Let X, Y, and Z denote the three different families in some order. Then the only possible arrangements are to have the second row be members of XYZ and the third row be members of ZXY, or to have the second row be members of XYZ and the third row be members of YZX. Note that these are not the same, because in the first case one sibling pair occupy the right-most seat in the second row and the left-most seat in the third row, whereas in the second case this does not happen. (Having members of XYX in the second row does not work because then the third row must be members of ZYZ to avoid consecutive members of Z; but in this case one of the Y siblings would be seated directly in front of the other Y sibling.) In each of these 2 cases there are $3! = 6$ ways to assign the families to the letters and $2^3 = 8$ ways to position the boy and girl within the seats assigned to the families. Therefore the total number of seating arrangements is $2 \cdot 6 \cdot 8 = 96$.

19. **Answer (E):** Let Chloe be n years old today, so she is $n - 1$ years older than Zoe. For integers $y \geq 0$, Chloe's age will be a multiple of Zoe's age y years from now if and only if

$$\frac{n+y}{1+y} = 1 + \frac{n-1}{1+y}$$

is an integer, that is, $1 + y$ is a divisor of $n - 1$. Thus $n - 1$ has exactly 9 positive integer divisors, so the prime factorization of $n - 1$ has one of the two forms p^2q^2 or p^8. There are no two-digit integers of the form p^8, and the only one of the form p^2q^2 is $2^2 \cdot 3^2 = 36$. Therefore Chloe is 37 years old today, and Joey is 38. His age will be a multiple of Zoe's age in y years if and only if $1 + y$ is a divisor of $38 - 1 = 37$. The nonnegative integer solutions for y are 0 and 36, so the only other time Joey's age will be a multiple of Zoe's age will be when he is $38 + 36 = 74$ years old. The requested sum is $7 + 4 = 11$.

20. **Answer (B):** Applying the recursion for several steps leads to the conjecture that

$$f(n) = \begin{cases} n + 2 & \text{if } n \equiv 0 \pmod 6, \\ n & \text{if } n \equiv 1 \pmod 6, \\ n - 1 & \text{if } n \equiv 2 \pmod 6, \\ n & \text{if } n \equiv 3 \pmod 6, \\ n + 2 & \text{if } n \equiv 4 \pmod 6, \\ n + 3 & \text{if } n \equiv 5 \pmod 6. \end{cases}$$

The conjecture can be verified using the strong form of mathematical induction with two base cases and six inductive steps. For example, if $n \equiv 2 \pmod 6$, then $n = 6k + 2$ for some nonnegative integer k and

$$\begin{aligned} f(n) &= f(6k + 2) \\ &= f(6k + 1) - f(6k) + 6k + 2 \\ &= (6k + 1) - (6k + 2) + 6k + 2 \\ &= 6k + 1 \\ &= n - 1. \end{aligned}$$

Therefore $f(2018) = f(6 \cdot 336 + 2) = 2018 - 1 = 2017$.

OR

Note that

$$\begin{aligned} f(n) &= f(n - 1) - f(n - 2) + n \\ &= [f(n - 2) - f(n - 3) + (n - 1)] - f(n - 2) + n \\ &= -[f(n - 4) - f(n - 5) + (n - 3)] + 2n - 1 \\ &= -[f(n - 5) - f(n - 6) + (n - 4)] + f(n - 5) + n + 2 \\ &= f(n - 6) + 6. \end{aligned}$$

It follows that $f(2018) = f(2) + 2016 = 2017$.

21. **Answer (C):** Let d be the next divisor of n after 323. Then $\gcd(d, 323) \neq 1$, because otherwise $n \geq 323d > 323^2 > 100^2 = 10000$, contrary to the fact that n is a 4-digit number. Therefore $d - 323 \geq \gcd(d, 323) > 1$. The prime factorization of 323 is $17 \cdot 19$. Thus the next divisor of n is at least $323 + 17 = 340 = 17 \cdot 20$. Indeed, 340 will be the next number in Mary's list when $n = 17 \cdot 19 \cdot 20 = 6460$.

22. **Answer (C):** The set of all possible ordered pairs (x, y) occupies the unit square $0 \leq x \leq 1$, $0 \leq y \leq 1$ in the Cartesian plane. The numbers x, y, and 1 are the side lengths of a triangle if and only if $x + y > 1$, which means that (x, y) lies above the line $y = 1 - x$. By a generalization of the Pythagorean Theorem, the triangle is obtuse if and only if, in addition, $x^2 + y^2 < 1^2$, which means that (x, y) lies inside the circle of radius 1 centered at the origin. Within the unit square, the region inside the circle of radius 1 centered at the origin has area $\frac{\pi}{4}$, and the region below the line $y = 1 - x$ has area $\frac{1}{2}$. Therefore the ordered pairs that meet the required conditions occupy a region with area $\frac{\pi}{4} - \frac{1}{2} = \frac{\pi - 2}{4}$. The area of the unit square is 1, so the required probability is also $\frac{\pi - 2}{4} \approx \frac{1.14}{4} = 0.285$, which is closest to 0.29.
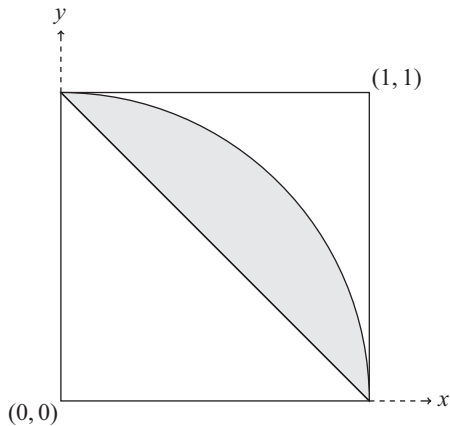
23. **Answer (B):** Recall that $a \cdot b = \gcd(a, b) \cdot \operatorname{lcm}(a, b)$. Let $x = \operatorname{lcm}(a, b)$ and $y = \gcd(a, b)$. The given equation is then $xy + 63 = 20x + 12y$, which can be rewritten as

$$(x - 12)(y - 20) = 240 - 63 = 177 = 3 \cdot 59 = 1 \cdot 177.$$

Because x and y are integers, one of the following must be true:

- $x - 12 = 1$ and $y - 20 = 177$,
- $x - 12 = 177$ and $y - 20 = 1$,
- $x - 12 = 3$ and $y - 20 = 59$,
- $x - 12 = 59$ and $y - 20 = 3$.

Therefore (x, y) must be $(13, 197)$, $(189, 21)$, $(15, 79)$, or $(71, 23)$. Because x must be a multiple of y, only $(x, y) = (189, 21)$ is possible. Therefore $\gcd(a, b) = 21 = 7 \cdot 3$, and $\text{lcm}(a, b) = 189 = 7 \cdot 3^3$. Both a and b are divisible by 7 but not by 7^2; one of a and b is divisible by 3 but not 3^2, and the other is divisible by 3^3 but not 3^4; and neither is divisible by any other prime. Therefore one of them is $7 \cdot 3 = 21$ and the other is $7 \cdot 3^3 = 189$. There are 2 ordered pairs, $(a, b) = (21, 189)$ and $(a, b) = (189, 21)$.

24. **Answer (C):**   Let O be the center of the regular hexagon. Points B, O, E are collinear and $BE = BO + OE = 2$. Trapezoid $FABE$ is isosceles, and $\overline{XZ}$ is its midline. Hence $XZ = \frac{3}{2}$ and analogously $XY = ZY = \frac{3}{2}$.
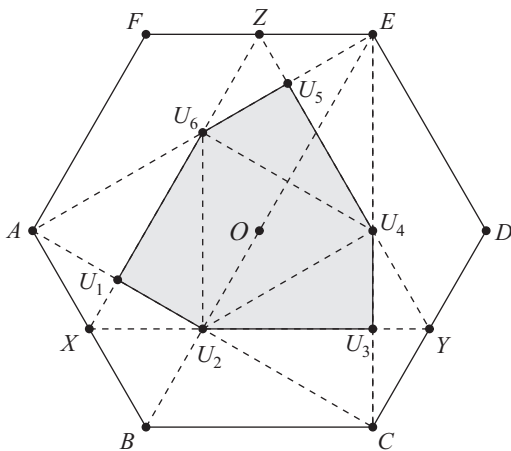
Denote by U_1 the intersection of $\overline{AC}$ and $\overline{XZ}$ and by U_2 the intersection of $\overline{AC}$ and $\overline{XY}$. It is easy to see that $\triangle AXU_1$ and $\triangle U_2XU_1$ are congruent $30 - 60 - 90°$ right triangles.

By symmetry the area of the convex hexagon enclosed by the intersection of $\triangle ACE$ and $\triangle XYZ$, shaded in the figure, is equal to the area of $\triangle XYZ$ minus 3 times the area of $\triangle U_2XU_1$. The hypotenuse

of $\triangle U_2XU_1$ is $XU_2 = AX = \frac{1}{2}$, so the area of $\triangle U_2XU_1$ is

$$\frac{1}{2} \cdot \frac{\sqrt{3}}{4} \cdot \left(\frac{1}{2}\right)^2 = \frac{1}{32}\sqrt{3}.$$

The area of the equilateral triangle XYZ with side length $\frac{3}{2}$ is equal to $\frac{1}{4}\sqrt{3} \cdot \left(\frac{3}{2}\right)^2 = \frac{9}{16}\sqrt{3}$. Hence the area of the shaded hexagon is

$$\frac{9}{16}\sqrt{3} - 3 \cdot \frac{1}{32}\sqrt{3} = 3\sqrt{3}\left(\frac{3}{16} - \frac{1}{32}\right) = \frac{15}{32}\sqrt{3}.$$

OR

Let U_1 and U_2 be as above, and continue labeling the vertices of the shaded hexagon counterclockwise with U_3, U_4, U_5, and U_6 as shown. The area of $\triangle ACE$ is half the area of hexagon $ABCDEF$. Triangle $U_2U_4U_6$ is the midpoint triangle of $\triangle ACE$, so its area is $\frac{1}{4}$ of the area of $\triangle ACE$, and thus $\frac{1}{8}$ of the area of $ABCDEF$. Each of $\triangle U_2U_3U_4$, $\triangle U_4U_5U_6$, and $\triangle U_6U_1U_2$ is congruent to half of $\triangle U_2U_4U_6$, so the total shaded area is $\frac{5}{2}$ times the area of $\triangle U_2U_4U_6$ and therefore $\frac{5}{2} \cdot \frac{1}{8} = \frac{5}{16}$ of the area of $ABCDEF$. The area of $ABCDEF$ is $6 \cdot \frac{\sqrt{3}}{4} \cdot 1^2$, so the requested area is $\frac{15}{32}\sqrt{3}$.

25. **Answer (C):** Let $\{x\} = x - \lfloor x \rfloor$ denote the fractional part of x. Then $0 \le \{x\} < 1$. The given equation is equivalent to $x^2 = 10{,}000\{x\}$, that is,

$$\frac{x^2}{10{,}000} = \{x\}.$$

Therefore if x satisfies the equation, then

$$0 \le \frac{x^2}{10{,}000} < 1.$$

This implies that $x^2 < 10{,}000$, so $-100 < x < 100$. The figure shows a sketch of the graphs of

$$f(x) = \frac{x^2}{10{,}000} \qquad \text{and} \qquad g(x) = \{x\}$$

for $-100 < x < 100$ on the same coordinate axes. The graph of g consists of the 200 half-open line segments with slope 1 connecting the points $(k, 0)$ and $(k+1, 1)$ for $k = -100, -99, \ldots, 98, 99$. (The

endpoints of these intervals that lie on the x-axis are part of the graph, but the endpoints with y-coordinate 1 are not.) It is clear that there is one intersection point for x lying in each of the intervals $[-100, -99)$, $[-99, -98)$, $[-98, -97)$, ..., $[-1, 0)$, $[0, 1)$, $[1, 2)$, ..., $[97, 98)$, $[98, 99)$ but no others. Thus the equation has 199 solutions.
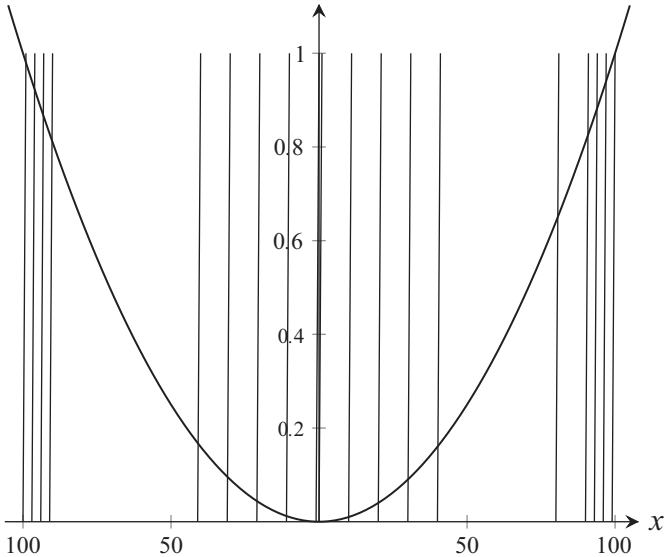
OR

The solutions to the equation correspond to points of intersection of the graphs $y = 10000\lfloor x \rfloor$ and $y = 10000x - x^2$. There will be a point of intersection any time the parabola intersects the half-open horizontal segment from the point $(a, 10000a)$ to the point $(a+1, 10000a)$, where a is an integer. This occurs for every integer value of a for which

$$10000a - a^2 \leq 10000a < 10000(a + 1) - (a + 1)^2.$$

This is equivalent to $(a + 1)^2 < 10000$, which occurs if and only if $-101 < a < 99$. Thus points of intersection occur on the intervals $[a, a + 1)$ for $a = -100, -99, -98, \ldots, -1, 0, 1, \ldots, 97, 98$, resulting in 199 points of intersection.