15

ARITHMETIC

2002A

1. **(D)** We have

$$\frac{10^{2000}+10^{2002}}{10^{2001}+10^{2001}}=\frac{10^{2000}(1+100)}{10^{2000}(10+10)}=\frac{101}{20}\approx 5.$$

2002B

1. **(E)** We have

$$\frac{2^{2001}\cdot 3^{2003}}{6^{2002}}=\frac{2^{2001}\cdot 3^{2003}}{(2\cdot 3)^{2002}}=\frac{2^{2001}\cdot 3^{2003}}{2^{2002}\cdot 3^{2002}}=\frac{3}{2}.$$

2003B

1. (C) We have

$$\frac{2-4+6-8+10-12+14}{3-6+9-12+15-18+21}=\frac{2(1-2+3-4+5-6+7)}{3(1-2+3-4+5-6+7)}=\frac{2}{3}.$$

2006B 1. **(C)** Because

$$(-1)^k = \begin{cases} 1, & \text{if } k \text{ is even,} \\ -1, & \text{if } k \text{ is odd,} \end{cases}$$

the sum can be written as

$$(-1+1)+(-1+1)+\cdots+(-1+1) = 0+0+\cdots+0 = 0.$$

2010B 1. **Answer (C):** Simplifying gives

$$\begin{aligned} 100(100-3)-(100\cdot 100-3) &= 100\cdot 100 - 100\cdot 3 - 100\cdot 100 + 3 \\ &= -300+3 \\ &= -297. \end{aligned}$$

2015A 1. **Answer (C):**

$$(1-1+25+0)^{-1}\times 5 = \frac{1}{25}\times 5 = \frac{1}{5}$$

2015B 1. **Answer (C):**

$$2-(-2)^{-2} = 2-\frac{1}{(-2)^2} = 2-\frac{1}{4} = \frac{7}{4}$$

2017A 1. **Answer (C):**

$$\begin{aligned}(2(2(2(2(2(2+1)+1)+1)+1)+1)+1)\\ &= (2(2(2(2(2(3)+1)+1)+1)+1)+1)\\ &= (2(2(2(2(7)+1)+1)+1)+1)\\ &= (2(2(2(15)+1)+1)+1)\\ &= (2(2(31)+1)+1)\\ &= (2(63)+1)\\ &= 127\end{aligned}$$

Observe that each intermediate result is 1 less than a power of 2.

2016A

1. **Answer (B):**

$$\frac{11!-10!}{9!} = \frac{10!\cdot(11-1)}{9!} = \frac{10\cdot 9!\cdot 10}{9!} = 100$$

2018A 1. **Answer (B):** Computing inside to outside yields:

$$\begin{aligned}\left(\left((2+1)^{-1}+1\right)^{-1}+1\right)^{-1}+1 &= \left(\left(\frac{4}{3}\right)^{-1}+1\right)^{-1}+1\\ &= \left(\frac{7}{4}\right)^{-1}+1\\ &= \frac{11}{7}.\end{aligned}$$

Note: The successive denominators and numerators of numbers obtained from this pattern are the Lucas numbers.

2002B

2. **(C)** We have

$$(2,4,6) = \frac{2\cdot 4\cdot 6}{2+4+6} = \frac{48}{12} = 4.$$

2000

2. **Answer (A):** $2000(2000^{2000}) = (2000^1)(2000^{2000}) = 2000^{1+2000} = 2000^{2001}$. All the other options are greater than 2000^{2001}.

2008B

3. **Answer (D):** The properties of exponents imply that

$$\sqrt[3]{x\sqrt{x}} = \left(x \cdot x^{\frac{1}{2}}\right)^{\frac{1}{3}} = \left(x^{\frac{3}{2}}\right)^{\frac{1}{3}} = x^{\frac{1}{2}}.$$

2016B

3. **Answer (D):**

$$\Bigg|\,\big|\,|-2016|-(-2016)\,\big| - |-2016|\,\Bigg| - (-2016)$$
$$= \Bigg|\,\big|\,2016+2016\,\big| - 2016\,\Bigg| + 2016 = 2016 + 2016 = 4032$$

2002B

4. **(D)** Since

$$(3x-2)(4x+1)-(3x-2)4x+1 = (3x-2)(4x+1-4x)+1$$
$$= (3x-2)\cdot 1 + 1 = 3x-1,$$

when $x = 4$ we have the value $3 \cdot 4 - 1 = 11$.

2011A

4. **Answer (A):** Every term in X except 10 appears in Y. Every term in Y except 102 appears in X. Therefore $Y - X = 102 - 10 = 92$.

OR

The sum X has 46 terms because it includes all 50 even positive integers less than or equal to 100 except for 2, 4, 6, and 8. The sum Y has the same number of terms, and every term in Y exceeds the corresponding term in X by 2. Therefore $Y - X = 46 \cdot 2 = 92$.

2009A

5. **Answer (E):** The square of 111,111,111 is

$$
\begin{array}{ccccccccccccccccc}
&&&&&&&&1&1&1&1&1&1&1&1&1\\
&&&&&&&\times&1&1&1&1&1&1&1&1&1\\
\hline
&&&&&&&&1&1&1&1&1&1&1&1&1\\
&&&&&&&1&1&1&1&1&1&1&1&1&\\
&&&&&&1&1&1&1&1&1&1&1&1&&\\
&&&&&1&1&1&1&1&1&1&1&1&&&\\
&&&&1&1&1&1&1&1&1&1&1&&&&\\
&&&1&1&1&1&1&1&1&1&1&&&&&\\
&&1&1&1&1&1&1&1&1&1&&&&&&\\
&1&1&1&1&1&1&1&1&1&&&&&&&\\
1&1&1&1&1&1&1&1&1&&&&&&&&\\
\hline
1&2&3&4&5&6&7&8&9&8&7&6&5&4&3&2&1
\end{array}
$$

Hence the sum of the digits of the square of 111,111,111 is 81.