16

SOLVE FOR X

2016A 2. **Answer (C):** The equation can be written $10^x \cdot (10^2)^{2x} = (10^3)^5$ or $10^x \cdot 10^{4x} = 10^{15}$. Thus $10^{5x} = 10^{15}$, so $5x = 15$ and $x = 3$.

2004A 2. **(B)** Because

$$\P(1,2,3) = \frac{1}{2-3} = -1, \quad \P(2,3,1) = \frac{2}{3-1} = 1, \text{ and}$$
$$\P(3,1,2) = \frac{3}{1-2} = -3,$$

we have

$$\begin{aligned}\P\left(\P(1,2,3), \P(2,3,1), \P(3,1,2)\right) &= \P(-1,1,-3) \\ &= \frac{-1}{1-(-3)} = -\frac{1}{4}.\end{aligned}$$

2005A

2. **(C)** First we have

$$(1 \star 2) = \frac{1+2}{1-2} = -3.$$

Then

$$((1 \star 2) \star 3) = (-3 \star 3) = \frac{-3+3}{-3-3} = \frac{0}{-6} = 0.$$

2006A

2. **(C)** By the definition we have

$$h \otimes (h \otimes h) = h \otimes (h^3 - h) = h^3 - (h^3 - h) = h.$$

2006B

2. **(A)** Because $4 \spadesuit 5 = (4+5)(4-5) = -9$, it follows that

$$3 \spadesuit (4 \spadesuit 5) = 3 \spadesuit (-9) = (3 + (-9))(3 - (-9)) = (-6)(12) = -72.$$

2007A

2. **Answer (A):** The value of $6@2$ is $6 \cdot 2 - 2^2 = 12 - 4 = 8$, and the value of $6\#2$ is $6 + 2 - 6 \cdot 2^2 = 8 - 24 = -16$. Thus

$$\frac{6@2}{6\#2} = \frac{8}{-16} = -\frac{1}{2}.$$

2007B

2. **Answer (E):** Since $3 \star 5 = (3+5)5 = 8 \cdot 5 = 40$ and $5 \star 3 = (5+3)3 = 8 \cdot 3 = 24$, we have

$$3 \star 5 - 5 \star 3 = 40 - 24 = 16.$$

2016B

2. **Answer (B):**

$$\frac{2\heartsuit 4}{4\heartsuit 2} = \frac{2^3 \cdot 4^2}{4^3 \cdot 2^2} = \frac{2}{4} = \frac{1}{2}$$

2004A

4. **(D)** The equation implies that either

$$x-1 = x-2 \quad \text{or} \quad x-1 = -(x-2)$$

The first equation has no solution, and the solution to the second equation is $x = 3/2$.

OR

Since $|x-a|$ is the distance of x from a, x must be equidistant from 1 and 2. Hence $x = 3/2$.

2005B

4. **(D)** It follows from the definition that

$$\begin{aligned}(5 \diamond 12) \diamond ((-12) \diamond (-5)) &= \sqrt{5^2+12^2} \diamond \sqrt{(-12)^2+(-5)^2} \\ &= 13 \diamond 13 = \sqrt{13^2+13^2} = 13\sqrt{2}.\end{aligned}$$

2010B

4. **Answer (C):** Note that $\heartsuit(1) = \frac{1+1^2}{2} = 1$, $\heartsuit(2) = \frac{2+2^2}{2} = 3$, and $\heartsuit(3) = \frac{3+3^2}{2} = 6$. Thus $\heartsuit(1) + \heartsuit(2) + \heartsuit(3) = 1+3+6 = 10$.

2017B

4. **Answer (D):** The given equation implies that $3x+y = -2(x-3y)$, which is equivalent to $x = y$. Therefore

$$\frac{x+3y}{3x-y} = \frac{4y}{2y} = 2.$$

2003A 5. (B) Since

$$0 = 2x^2 + 3x - 5 = (2x + 5)(x - 1) \quad \text{we have} \quad d = -\frac{5}{2} \text{ and } e = 1.$$

So $(d-1)(e-1) = 0$.

OR

If $x = d$ and $x = e$ are the roots of the quadratic equation $ax^2 + bx + c = 0$, then

$$de = \frac{c}{a} \quad \text{and} \quad d + e = -\frac{b}{a}.$$

For our equation this implies that

$$(d-1)(e-1) = de - (d+e) + 1 = -\frac{5}{2} - \left(-\frac{3}{2}\right) + 1 = 0.$$

2008B 5. **Answer (A):** Note that $(y-x)^2 = (x-y)^2$, so

$$(x-y)^2\$(y-x)^2 = (x-y)^2\$(x-y)^2 = \left((x-y)^2 - (x-y)^2\right)^2 = 0^2 = 0.$$