17

ALGEBRA WORD PROBLEMS

2004B

1. **(C)** There are $22 - 12 + 1 = 11$ reserved rows. Because there are 33 seats in each row, there are $(33)(11) = 363$ reserved seats.

2006A

1. **(A)** Five sandwiches cost $5 \cdot 3 = 15$ dollars and eight sodas cost $8 \cdot 2 = 16$ dollars. Together they cost $15 + 16 = 31$ dollars.

2017B

1. **Answer (B):** Working backwards, switching the digits of the numbers 71, 72, 73, 74, and 75 and subtracting 11 gives, respectively, 6, 16, 26, 36, and 46. Only 6 and 36 are divisible by 3, and only $36 \div 3 = 12$ is a two-digit number.

2001 2. **(C)** The reciprocal of x is $\frac{1}{x}$, and the additive inverse of x is $-x$. The product of these is $(\frac{1}{x}) \cdot (-x) = -1$. So $x = -1 + 2 = 1$, which is in the interval $0 < x \leq 2$.

2003B 3. (B) Let n be the smallest of the even integers. Since

$$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 = 64,$$

we have

$$60 = n + (n + 2) + (n + 4) + (n + 6) + (n + 8) = 5n + 20, \quad \text{so} \quad n = 8.$$

2002A 3. **(B)** No matter how the exponentiations are performed, 2^{2^2} always gives 16. Depending on which exponentiation is done last, we have

$$\left(2^{2^2}\right)^2 = 256, \quad 2^{\left(2^{2^2}\right)} = 65,536, \quad \text{or} \quad (2^2)^{(2^2)} = 256,$$

so there is one other possible value.

2012A 3. **Answer (E):** The distance from -2 to -6 is $|(-6) - (-2)| = 4$ units. The distance from -6 to 5 is $|5 - (-6)| = 11$ units. Altogether the bug crawls $4 + 11 = 15$ units.

2002A 4. **(E)** When $n = 1$, the inequality becomes $m \leq 1 + m$, which is satisfied by all integers m. Thus, there are infinitely many of the desired values of m.

2013B

4. **Answer (D):** The number 201 is the 1^{st} number counted when proceeding backwards from 201 to 3. In turn, 200 is the 2^{nd} number, 199 is the 3^{rd} number, and x is the $(202-x)^{\text{th}}$ number. Therefore 53 is the $(202-53)^{\text{th}}$ number, which is the 149^{th} number.

2014B

4. **Answer (B):** Let a muffin cost m dollars and a banana cost b dollars. Then $2(4m+3b) = 2m+16b$, and simplifying gives $m = \frac{5}{3}b$.

2016A 4. **Answer (B):**

$$\frac{3}{8} - \left(-\frac{2}{5}\right)\left\lfloor \frac{\frac{3}{8}}{-\frac{2}{5}} \right\rfloor = \frac{3}{8} + \frac{2}{5}\left\lfloor -\frac{15}{16} \right\rfloor = \frac{3}{8} + \frac{2}{5}(-1) = -\frac{1}{40}$$

2017A

4. **Answer (B):** After exactly half a minute there will be 3 toys in the box and 27 toys outside the box. During the next half-minute, Mia takes 2 toys out and her mom puts 3 toys into the box. This

means that during this half-minute the number of toys in the box was increased by 1. The same argument applies to each of the following half-minutes until all the toys are in the box for the first time. Therefore it takes $1 + 27 \cdot 1 = 28$ half-minutes, which is 14 minutes, to complete the task.

2018B

5. **Answer (D):** The number of qualifying subsets equals the difference between the total number of subsets of $\{2, 3, 4, 5, 6, 7, 8, 9\}$ and the number of such subsets containing no prime numbers, which is the number of subsets of $\{4, 6, 8, 9\}$. A set with n elements has 2^n subsets, so the requested number is $2^8 - 2^4 = 256 - 16 = 240$.

OR

A subset meeting the condition must be the union of a nonempty subset of $\{2, 3, 5, 7\}$ and a subset of $\{4, 6, 8, 9\}$. There are $2^4 - 1 = 15$ of the former and $2^4 = 16$ of the latter, which gives $15 \cdot 16 = 240$ choices in all.

2017A

5. **Answer (C):** Let the two numbers be x and y. Then $x + y = 4xy$. Dividing this equation by xy gives $\frac{1}{y} + \frac{1}{x} = 4$. One such pair of numbers is $x = \frac{1}{3}$, $y = 1$.

2011B

5. **Answer (E):** Because $161 = 23 \cdot 7$, the only two digit factor of 161 is 23. The correct product must have been $32 \cdot 7 = 224$.

2013B

5. **Answer (B):** Note that $2 \cdot a - a \cdot b = (2 - b)a$. This expression is negative when $b > 2$. Hence the product is minimized when a and b are as large as possible. The minimum value is $(2 - 5) \cdot 5 = -15$.

2001

3. **(E)** Suppose the two numbers are a and b. Then the desired sum is

$$2(a_3) + 2(b + 3) = 2(a + b) + 12 = 2S + 12.$$

2008A

3. **Answer (A):** The positive divisors of 6, other than 6, are 1, 2, and 3, so $<6> = 1 + 2 + 3 = 6$. As a consequence, we also have $<<<6>>> = 6$.

Note: A positive integer whose divisors other than itself add up to that positive integer is called a perfect number. The two smallest perfect numbers are 6 and 28.

2010A

3. **Answer (D):** Let x be the number of marbles that Tyrone gave to Eric. Then $97 - x = 2(11 + x)$. Solving this equation yields $x = 25$.

2015B

3. **Answer (A):** Let x be the integer Isaac wrote two times, and let y be the integer Isaac wrote three times. Then $2x + 3y = 100$. If $x = 28$, then $3y = 100 - 2 \cdot 28 = 44$, and y cannot be an integer. Therefore $y = 28$ and $2x = 100 - 3 \cdot 28 = 16$, so $x = 8$.

2017B

3. **Answer (E):** Adding the inequalities $y > -1$ and $z > 1$ yields $y + z > 0$. The other four choices give negative values if, for example, $x = \frac{1}{8}$, $y = -\frac{1}{4}$, and $z = \frac{3}{2}$.

2007A 4. **Answer (A):** Let the smaller of the integers be x. Then the larger is $x+2$. So $x+2=3x$, from which $x=1$. Thus the two integers are 1 and 3, and their sum is 4.

2013A 5. **Answer (B):** The total shared expenses were $105+125+175=405$ dollars, so each traveler's fair share was $\frac{1}{3}\cdot 405=135$ dollars. Therefore $t=135-105=30$ and $d=135-125=10$, so $t-d=30-10=20$.

OR

Because Dorothy paid 20 dollars more than Tom, Sammy must receive 20 more dollars from Tom than from Dorothy.

2017B 5. **Answer (D):** Suppose Camilla originally had b blueberry jelly beans and c cherry jelly beans. After eating 10 pieces of each kind, she now has $b-10$ blueberry jelly beans and $c-10$ cherry jelly beans. The conditions of the problem are equivalent to the equations $b=2c$ and $b-10=3(c-10)$. Then $2c-10=3c-30$, which means that $c=20$ and $b=2\cdot 20=40$.