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VENN DIAGRAM

2017B

13. **Answer (C):** Let x, y, and z be the number of people taking exactly one, two, and three classes, respectively. The condition that each student in the program takes at least one class is equivalent to the equation $x + y + z = 20$. The condition that there are 9 students taking at least two classes is equivalent to the equation $y + z = 9$. The sum $10 + 13 + 9 = 32$ counts once the students taking one class, twice the students taking two classes, and three times the students taking three classes. Then $x + 2y + 3z = 32$, which is equivalent to $z = 32 - (x + y + z) - (y + z) = 32 - 20 - 9 = 3$.

OR

Let Y, B, and P be the sets of students taking yoga, bridge, and painting, respectively. By the Inclusion–Exclusion Principle,

$$|Y \cup B \cup P| = |Y| + |B| + |P| - (|Y \cap B| + |Y \cap P| + |B \cap P|) + |Y \cap B \cap P|.$$

Furthermore, $|Y \cap B| + |Y \cap P| + |B \cap P| = 9 + 2|Y \cap B \cap P|$, because in tabulating the students taking at least two classes by considering the pairs of classes one by one, the students taking all three classes are counted three times rather than just once. Thus

$$20 = 10 + 13 + 9 - (9 + 2|Y \cap B \cap P|) + |Y \cap B \cap P| = 23 - |Y \cap B \cap P|,$$

so the number of students taking all three classes is $|Y \cap B \cap P| = 3$.