12

PERCENT

2004A 11. **(C)** Let r, h, and V, respectively, be the radius, height, and volume of the jar that is currently being used. The new jar will have a radius of $1.25r$ and volume V. Let H be the height of the new jar. Then

$$\pi r^2 h = V = \pi(1.25r)^2 H, \quad \text{so} \quad \frac{H}{h} = \frac{1}{(1.25)^2} = 0.64.$$

Thus H is 64% of h, so the height must be reduced by $(100 - 64)\% = 36\%$.

OR

Multiplying the diameter by 5/4 multiplies the area of the base by $(5/4)^2 = 25/16$, so in order to keep the same volume, the height must be multiplied by 16/25. Thus the height must be decreased by 9/25, or 36%.

2010B 11. **Answer (A):** Let p dollars be the purchase price of the stem. The savings provided by Coupon A, B, and C respectively are $0.15p$, 30, and $0.25(p - 100)$. Coupon A saves at least as much as Coupon B if $0.15p \geq 30$, so $p \geq 200$. Coupon A saves at least as much as Coupon C if $0.15p \geq 0.25(p - 100)$, so $p \leq 250$. Therefore $x = 200$, $y = 250$, and $y - x = 50$.

2014A

11. **Answer (C):** Let $P > 100$ be the listed price. Then the price reductions in dollars are as follows:

Coupon 1: $\frac{P}{10}$
Coupon 2: 20
Coupon 3: $\frac{18}{100}(P - 100)$

Coupon 1 gives a greater price reduction than coupon 2 when $\frac{P}{10} > 20$, that is, $P > 200$. Coupon 1 gives a greater price reduction than coupon 3 when $\frac{P}{10} > \frac{18}{100}(P - 100)$, that is, $P < 225$. The only choice that satisfies these inequalities is \$219.95.

2014B

11. **Answer (C):** If P is the price paid for an item, then the discounted prices with the three given discounts are given by the following calculations:

(1) $(0.85)^2P = 0.7225P$ for a discount of 27.75%

(2) $(0.9)^3P = 0.729P$ for a discount of 27.1%

(3) $(0.75) \cdot (0.95)P = 0.7125P$ for a discount of 28.75%

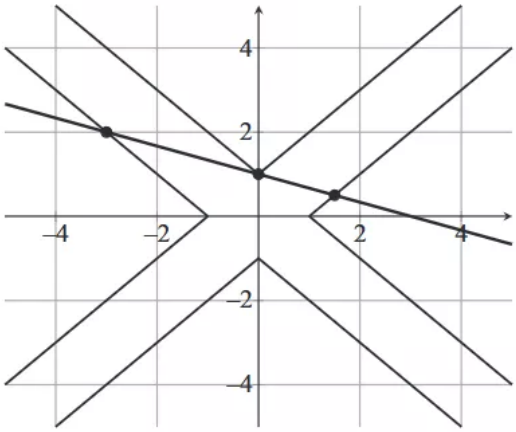
The smallest integer greater than 27.75, 27.1, and 28.75 is 29.

2017B

11. **Answer (D):** The students who like dancing but say they dislike it constitute $60\% \cdot (100\% - 80\%) = 12\%$ of the students. Similarly, the students who dislike dancing and say they dislike it constitute $(100\% - 60\%) \cdot 90\% = 36\%$ of the students. Therefore the requested fraction is $\frac{12}{12+36} = \frac{1}{4} = 25\%$.

2018A

12. **Answer (C):** The graph of the system is shown below.



The graph of the first equation is a line with x-intercept $(3,0)$ and y-intercept $(0,1)$. To draw the graph of the second equation, consider the equation quadrant by quadrant. In the first quadrant $x > 0$ and $y > 0$, and thus the second equation is equivalent to $|x-y| = 1$, which in turn is equivalent to $y = x \pm 1$. Its graph consists of the rays with endpoints $(0,1)$ and $(1,0)$, as shown. In the second quadrant $x < 0$ and $y > 0$. The corresponding graph is the reflection of the first quadrant graph across the y-axis. The rest of the graph can be sketched by further reflections of the first-quadrant graph across the coordinate axes, resulting in the figure shown. There are 3 intersection points: $(-3,2)$, $(0,1)$, and $\left(\frac{3}{2},\frac{1}{2}\right)$, as shown.

OR

The second equation implies that $x = y \pm 1$ or $x = -y \pm 1$. There are four cases:

- If $x = y + 1$, then $(y+1) + 3y = 3$, so $(x,y) = \left(\frac{3}{2},\frac{1}{2}\right)$.
- If $x = y - 1$, then $(y-1) + 3y = 3$, so $(x,y) = (0,1)$.
- If $x = -y + 1$, then $(-y+1) + 3y = 3$, so again $(x,y) = (0,1)$.
- If $x = -y - 1$, then $(-y-1) + 3y = 3$, so $(x,y) = (-3,2)$.

It may be checked that each of these ordered pairs actually satisfies the given equations, so the total number of solutions is 3

2010B

12. **Answer (D):** Assume there are 100 students in Mr. Wells' class. Then at least $70 - 50 = 20$ students answered "No" at the beginning of the school year and "Yes" at the end, so $x \geq 20$. Because only 30 students answered "No" at the end of the school year, at least $50-30 = 20$ students who answered "Yes"' at the beginning of the year gave the same answer at the end, so $x \leq 80$. The difference between the maximum and minimum possible values of x is $80 - 20 = 60$. The minimum $x = 20$ is achieved if exactly 20 students answered "No" at the beginning and "Yes" at the end of the school year. The maximum $x = 80$ is achieved if exactly 20 students answered "Yes at the beginning and the end.

2017B

12. **Answer (A):** For Elmer's old car, let M be the fuel efficiency in kilometers per liter, and let C be the cost of fuel in dollars per liter. Then for his new car, the fuel efficiency is $1.5M$, and the cost of fuel is $1.2C$. The cost in dollars per kilometer for the old car is $\frac{C}{M}$, and for the new car it is $\frac{1.2C}{1.5M} = 0.8\frac{C}{M}$. Therefore, fuel for the long trip will cost 20% less in Elmer's new car.

2007B

14. **Answer (C):** Let g be the number of girls and b the number of boys initially in the group. Then $g = 0.4(g+b)$. After two girls leave and two boys arrive, the size of the entire group is unchanged, so $g-2 = 0.3(g+b)$. The solution of the system of equations

$$g = 0.4(g+b) \quad \text{and} \quad g-2 = 0.3(g+b)$$

is $g = 8$ and $b = 12$, so there were initially 8 girls.

OR

After two girls leave and two boys arrive, the size of the group is unchanged. So the two girls who left represent $40\% - 30\% = 10\%$ of the group. Thus the size of the group is 20, and the original number of girls was 40% of 20, or 8.

2009B

14. **Answer (D):** On Monday, day 1, the birds find $\frac{1}{4}$ quart of millet in the feeder. On Tuesday they find

$$\frac{1}{4} + \frac{3}{4} \cdot \frac{1}{4}$$

quarts of millet. On Wednesday, day 3, they find

$$\frac{1}{4} + \frac{3}{4} \cdot \frac{1}{4} + \left(\frac{3}{4}\right)^2 \cdot \frac{1}{4}$$

quarts of millet. The number of quarts of millet they find on day n is

$$\frac{1}{4} + \frac{3}{4} \cdot \frac{1}{4} + \left(\frac{3}{4}\right)^2 \cdot \frac{1}{4} + \cdots + \left(\frac{3}{4}\right)^{n-1} \cdot \frac{1}{4} = \frac{(\frac{1}{4})(1-(\frac{3}{4})^n)}{1-\frac{3}{4}} = 1 - \left(\frac{3}{4}\right)^n.$$

The birds always find $\frac{3}{4}$ quart of other seeds, so more than half the seeds are millet if $1-(\frac{3}{4})^n > \frac{3}{4}$, that is, when $(\frac{3}{4})^n < \frac{1}{4}$. Because $(\frac{3}{4})^4 = \frac{81}{256} > \frac{1}{4}$ and $(\frac{3}{4})^5 = \frac{243}{1024} < \frac{1}{4}$, this will first occur on day 5 which is Friday.

2017A

14. **Answer (D):** Let M be the cost of Roger's movie ticket, and let S be the cost of Roger's soda. Then $M = 0.20(A-S)$ and $S = 0.05(A-M)$. Thus $5M + S = A$ and $M + 20S = A$. Solving the system for M and S in terms of A gives $M = \frac{19}{99}A$ and $S = \frac{4}{99}A$. The total cost of the movie ticket and soda as a fraction of A is $\frac{23}{99} = 0.2323\ldots \approx 23\%$.