16

SOLVE FOR X

2003A

11. (E) Since the last two digits of $AMC10$ and $AMC12$ sum to 22, we have

$$AMC + AMC = 2(AMC) = 1234.$$

Hence $AMC = 617$, so $A = 6$, $M = 1$, $C = 7$, and $A + M + C = 6 + 1 + 7 = 14$.

2010A

11. **Answer (D):** The solution of the inequality is

$$\frac{a-3}{2} \le x \le \frac{b-3}{2}.$$

If $\frac{b-3}{2} - \frac{a-3}{2} = 10$, then $b - a = 20$.

2002B

13. **(D)** The given equation can be factored as

$$0 = 8xy - 12y + 2x - 3 = 4y(2x - 3) + (2x - 3) = (4y + 1)(2x - 3).$$

For this equation to be true for all values of $y$ we must have $2x - 3 = 0$, that is, $x = 3/2$.

2005A

13. **(E)** The condition is equivalent to

$$130n > n^2 > 2^4 = 16, \quad \text{so} \quad 130n > n^2 \text{ and } n^2 > 16.$$

This implies that $130 > n > 4$. So $n$ can be any of the 125 integers strictly between 130 and 4.

2009A

13. **Answer (E):** Note that

$$12^{mn} = (2^2 \cdot 3)^{mn} = 2^{2mn} \cdot 3^{mn} = (2^m)^{2n} \cdot (3^n)^m = P^{2n}Q^m.$$

Remark: The pair of integers $(2, 1)$ shows that the other choices are not possible.

2010B

13. **Answer (C):** If $60 - 2x > 0$, then $|2x - |60 - 2x|| = |4x - 60|$. Solving $x = 4x - 60$, and $x = -(4x - 60)$ results in $x = 20$, and $x = 12$, respectively, both of which satisfy the original equation.
If $60 - 2x < 0$, then $|2x - |60 - 2x|| = |2x + 60 - 2x| = 60$. Note that $x = 60$ satisfies the original equation. The sum of the solutions is $12 + 20 + 60 = 92$.

2014B

12. **Answer (C):** By inspection, the five smallest positive divisors of 2,014,000,000 are 1, 2, 4, 5, and 8. Therefore the fifth largest divisor is $\frac{2{,}014{,}000{,}000}{8} = 251{,}750{,}000$.

2003B

14. (D) Since $a$ must be divisible by 5, and $3^8 \cdot 5^2$ is divisible by $5^2$, but not by $5^3$, we have $b \le 2$. If $b = 1$, then

$$a^b = (3^8 5^2)^1 = (164,025)^1 \quad \text{and} \quad a + b = 164,026.$$

If $b = 2$, then

$$a^b = (3^4 5)^2 = 405^2 \quad \text{so} \quad a + b = 407,$$

which is the smallest value.

2018B

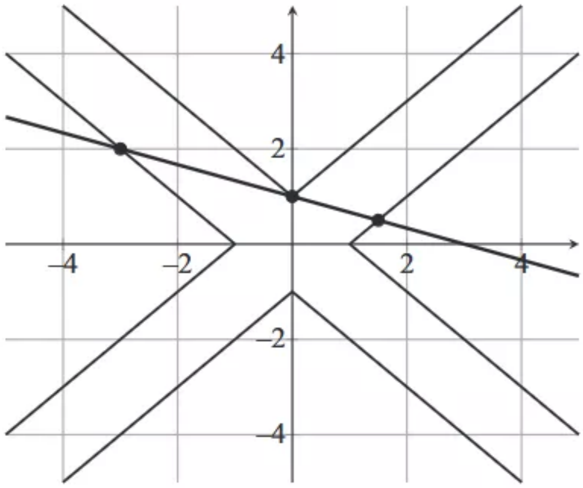
11. **Answer (C):** If $p = 3$, then $p^2 + 26 = 35 = 5 \cdot 7$. If $p$ is a prime number other than 3, then $p = 3k \pm 1$ for some positive integer $k$. In that case

$$p^2 + 26 = (3k \pm 1)^2 + 26 = 9k^2 \pm 6k + 27 = 3(3k^2 \pm 2k + 9)$$

is a multiple of 3 and is not prime. The smallest counterexamples for the other choices are $5^2 + 16 = 41$, $7^2 + 24 = 73$, $5^2 + 46 = 71$, and $19^2 + 96 = 457$.

2018A

12. **Answer (C):** The graph of the system is shown below.




The graph of the first equation is a line with $x$-intercept $(3,0)$ and $y$-intercept $(0,1)$. To draw the graph of the second equation, consider the equation quadrant by quadrant. In the first quadrant $x > 0$ and $y > 0$, and thus the second equation is equivalent to $|x-y| = 1$, which in turn is equivalent to $y = x \pm 1$. Its graph consists of the rays with endpoints $(0,1)$ and $(1,0)$, as shown. In the second quadrant $x < 0$ and $y > 0$. The corresponding graph is the reflection of the first quadrant graph across the $y$-axis. The rest of the graph can be sketched by further reflections of the first-quadrant graph across the coordinate axes, resulting in the figure shown. There are 3 intersection points: $(-3,2)$, $(0,1)$, and $\left(\frac{3}{2},\frac{1}{2}\right)$, as shown.

**OR**

The second equation implies that $x = y \pm 1$ or $x = -y \pm 1$. There are four cases:

- If $x = y + 1$, then $(y+1) + 3y = 3$, so $(x,y) = \left(\frac{3}{2},\frac{1}{2}\right)$.
- If $x = y - 1$, then $(y-1) + 3y = 3$, so $(x,y) = (0,1)$.
- If $x = -y + 1$, then $(-y+1) + 3y = 3$, so again $(x,y) = (0,1)$.
- If $x = -y - 1$, then $(-y-1) + 3y = 3$, so $(x,y) = (-3,2)$.

It may be checked that each of these ordered pairs actually satisfies the given equations, so the total number of solutions is 3

2000 15. **Answer (E):** Find the common denominator and replace the $ab$ in the numerator with $a-b$ to get

$$\begin{aligned}\frac{a}{b}+\frac{b}{a}-ab &= \frac{a^2+b^2-(ab)^2}{ab}\\ &= \frac{a^2+b^2-(a-b)^2}{ab}\\ &= \frac{a^2+b^2-(a^2-2ab+b^2)}{ab}\\ &= \frac{2ab}{ab}=2.\end{aligned}$$

**OR**

Note that $a=a/b-1$ and $b=1-b/a$. It follows that $\frac{a}{b}+\frac{b}{a}-ab=(a+1)+(1-b)-(a-b)=2$.

2004A 15. **(D)** Because

$$\frac{x+y}{x}=1+\frac{y}{x} \quad \text{and} \quad \frac{y}{x}<0,$$

the value is maximized when $|y/x|$ is minimized, that is, when $|y|$ is minimized and $|x|$ is maximized. So $y=2$ and $x=-4$ gives the largest value, which is $1+(-1/2)=1/2$.