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RATIO

2015A 11. **Answer (C):** Let the sides of the rectangle have lengths $3a$ and $4a$. By the Pythagorean Theorem, the diagonal has length $5a$. Because $5a = d$, the side lengths are $\frac{3}{5}d$ and $\frac{4}{5}d$. Therefore the area is $\frac{3}{5}d \cdot \frac{4}{5}d = \frac{12}{25}d^2$, so $k = \frac{12}{25}$.

2010A 12. **Answer (C):** The volume scale for Logan's model is $0.1 : 100{,}000 = 1 : 1{,}000{,}000$. Therefore the linear scale is $1 : \sqrt[3]{1{,}000{,}000}$, which is $1 : 100$. Logan's water tower should stand $\frac{40}{100} = 0.4$ meters tall.

2006B 13. **(E)** Joe has 2 ounces of cream in his cup. JoAnn has drunk 2 ounces of the 14 ounces of coffee-cream mixture in her cup, so she has only $12/14 = 6/7$ of her 2 ounces of cream in her cup. Therefore the ratio of the amount of cream in Joe's coffee to that in JoAnn's coffee is

$$\frac{2}{\frac{6}{7} \cdot 2} = \frac{7}{6}.$$

2007A

13. **Answer (B):** Let w be Yan's walking speed, and let x and y be the distances from Yan to his home and to the stadium, respectively. The time required for Yan to walk to the stadium is y/w, and the time required for him to walk home is x/w. Because he rides his bicycle at a speed of $7w$, the time required for him to ride his bicycle from his home to the stadium is $(x+y)/(7w)$. Thus

$$\frac{y}{w} = \frac{x}{w} + \frac{x+y}{7w} = \frac{8x+y}{7w}.$$

As a consequence, $7y = 8x + y$, so $8x = 6y$. The required ratio is $x/y = 6/8 = 3/4$.

OR

Because we are interested only in the ratio of the distances, we may assume that the distance from Yan's home to the stadium is 1 mile. Let x be his present distance from his home. Imagine that Yan has a twin, Nay. While Yan walks to the stadium, Nay walks to their home and continues $1/7$ of a mile past their home. Because walking $1/7$ of a mile requires the same amount of time as riding 1 mile, Yan and Nay will complete their trips at the same time. Yan has walked $1-x$ miles while Nay has walked $x + \frac{1}{7}$ miles, so $1 - x = x + \frac{1}{7}$. Thus $x = 3/7$, $1 - x = 4/7$, and the required ratio is $x/(1-x) = 3/4$.

2007A

14. **Answer (A):** Let the sides of the triangle have lengths $3x$, $4x$, and $5x$. The triangle is a right triangle, so its hypotenuse is a diameter of the circle. Thus $5x = 2 \cdot 3 = 6$, so $x = 6/5$. The area of the triangle is

$$\frac{1}{2} \cdot 3x \cdot 4x = \frac{1}{2} \cdot \frac{18}{5} \cdot \frac{24}{5} = \frac{216}{25} = 8.64.$$

OR

A right triangle with side lengths 3, 4, and 5 has area $(1/2)(3)(4) = 6$. Because the given right triangle is inscribed in a circle with diameter 6, the hypotenuse of this triangle has length 6. Thus the sides of the given triangle are $6/5$ as long as those of a $3-4-5$ triangle, and its area is $(6/5)^2$ times that of a $3-4-5$ triangle. The area of the given triangle is

$$\left(\frac{6}{5}\right)^2 (6) = \frac{216}{25} = 8.64.$$

2008A

14. **Answer (D):** Let h and w be the height and width of the screen, respectively, in inches. By the Pythagorean Theorem, $h{:}w{:}27 = 3{:}4{:}5$, so

$$h = \frac{3}{5} \cdot 27 = 16.2 \quad \text{and} \quad w = \frac{4}{5} \cdot 27 = 21.6.$$

The height of the non-darkened portion of the screen is half the width, or 10.8 inches. Therefore the height of each darkened strip is

$$\frac{1}{2}(16.2 - 10.8) = 2.7 \quad \text{inches.}$$

OR

The screen has dimensions $4a \times 3a$ for some a. The portion of the screen not covered by the darkened strips has aspect ratio 2:1, so it has dimensions $4a \times 2a$. Thus the darkened strips each have height $\frac{a}{2}$. By the Pythagorean Theorem, the diagonal of the screen is $5a = 27$ inches. Hence the height of each darkened strip is $\frac{27}{10} = 2.7$ inches.