2

3D GEOMTERY

2005A 11. **(B)** The unit cubes have a total of $6n^3$ faces, of which $6n^2$ are red. Therefore

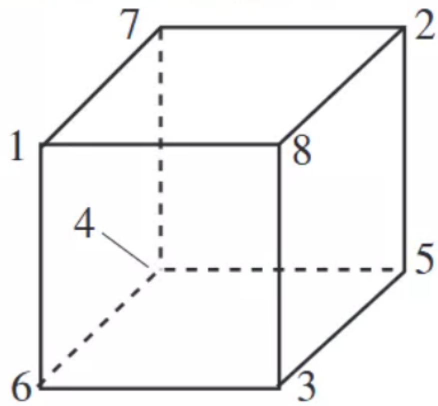
$$\frac{1}{4} = \frac{6n^2}{6n^3} = \frac{1}{n}, \quad \text{so} \quad n = 4.$$

2007A

11. **Answer (C):** Each vertex appears on exactly three faces, so the sum of the numbers on all the faces is

$$3(1+2+\cdots+8) = 3\cdot\frac{8\cdot 9}{2} = 108.$$

There are six faces for the cube, so the common sum must be $108/6 = 18$. A possible numbering is shown in the figure.




2009A

11. **Answer (D):** Let $x$ be the side length of the cube. Then the volume of the cube was $x^3$, and the volume of the new solid is $x(x+1)(x-1) = x^3 - x$. Therefore $x^3 - x = x^3 - 5$, from which $x = 5$, and the volume of the cube was $5^3 = 125$.

2017A

11. **Answer (D):** Let $h = AB$. The region consists of a solid circular cylinder of radius 3 and height $h$, together with two solid hemispheres of radius 3 centered at $A$ and $B$. The volume of the cylinder is $\pi\cdot 3^2\cdot h = 9\pi h$, and the two hemispheres have a combined volume of $\frac{4}{3}\pi\cdot 3^3 = 36\pi$. Therefore $9\pi h + 36\pi = 216\pi$, and $h = 20$.

2013A 14. **Answer (D):** The large cube has 12 edges, and a portion of each edge remains after the 8 small cubes are removed. All of the 12 edges of each small cube are also edges of the new solid, except for the 3 edges that meet at a vertex of the large cube. Thus the new solid has a total of $12 + 8(12 - 3) = 84$ edges.