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SEQUENCE SERIES

2005B

11. **(E)** The sequence begins 2005, 133, 55, 250, 133, ... Thus after the initial term 2005, the sequence repeats the cycle 133, 55, 250. Because $2005 = 1 + 3 \cdot 668$, the 2005^{th} term is the same as the last term of the repeating cycle, 250.

2008B

11. **Answer (B):** Note that $u_5 = 2u_4 + 9$ and $128 = u_6 = 2u_5 + u_4 = 5u_4 + 18$. Thus $u_4 = 22$, and it follows that $u_5 = 2 \cdot 22 + 9 = 53$.

2008B

13. **Answer (B):** Because the mean of the first n terms is n, their sum is n^2. Therefore the nth term is $n^2 - (n-1)^2 = 2n - 1$, and the 2008th term is $2 \cdot 2008 - 1 = 4015$.

2013B

13. **Answer (E):** Note that Jo starts by saying 1 number, and this is followed by Blair saying 2 numbers, then Jo saying 3 numbers, and so on. After someone completes her turn after saying the number n, then $1+2+3+\cdots+n = \frac{1}{2}n(n+1)$ numbers have been said. If $n = 9$ then 45 numbers have been said. Therefore there are $53 - 45 = 8$ more numbers that need to be said. The 53^{rd} number said is 8.

2017A

13. **Answer (D):** The sequence starts $0, 1, 1, 2, 0, 2, 2, 1, 0, 1, 1, 2, \ldots$. Notice that the pattern repeats and the period is 8. Thus no matter which 8 consecutive numbers are added, the answer will be $0 + 1 + 1 + 2 + 0 + 2 + 2 + 1 = 9$.

2009A

15. **Answer (E):** The outside square for F_n has 4 more diamonds on its boundary than the outside square for F_{n-1}. Because the outside square of F_2 has 4 diamonds, the outside square of F_n has $4(n-2)+4 = 4(n-1)$ diamonds. Hence the number of diamonds in figure F_n is the number of diamonds in F_{n-1} plus $4(n-1)$, or

$$\begin{aligned}
&1 + 4 + 8 + 12 + \cdots + 4(n-2) + 4(n-1) \\
=&1 + 4(1 + 2 + 3 + \cdots + (n-2) + (n-1)) \\
=&1 + 4\frac{(n-1)n}{2} \\
=&1 + 2(n-1)n.
\end{aligned}$$

Therefore figure F_{20} has $1 + 2 \cdot 19 \cdot 20 = 761$ diamonds.