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QUADRATICS

2006A

11. **(C)** The equation $(x+y)^2 = x^2 + y^2$ is equivalent to $x^2 + 2xy + y^2 = x^2 + y^2$, which reduces to $xy = 0$. Thus the graph of the equation consists of the two lines that are the coordinate axes.

2013B

11. **Answer (B):** By completing the square the equation can be rewritten as follows:

$$x^2 + y^2 = 10x - 6y - 34,$$
$$x^2 - 10x + 25 + y^2 + 6y + 9 = 0,$$
$$(x-5)^2 + (y+3)^2 = 0.$$

Therefore $x = 5$ and $y = -3$, so $x + y = 2$.

2002B

12. **(E)** From the given equation we have $(x-1)(x-6) = (x-2)(x-k)$. This implies that

$$x^2 - 7x + 6 = x^2 - (2+k)x + 2k,$$

so

$$(k-5)x = 2k - 6 \quad \text{and} \quad x = \frac{2k-6}{k-5}.$$

Hence a value of x satisfying the equation occurs unless $k = 5$.
Note that when $k = 6$ there is also no solution for x, but this is not one of the answer choices.

2015A

12. **Answer (C):** The equation is equivalent to $1 = y^2 - 2x^2y + x^4 = (y - x^2)^2$, or $y - x^2 = \pm 1$. The graph consists of two parabolas, $y = x^2 + 1$ and $y = x^2 - 1$. Thus a and b are $\pi + 1$ and $\pi - 1$, and their difference is 2. Indeed, the answer would still be 2 if $\sqrt{\pi}$ were replaced by any real number.

2006B

14. **(D)** Since a and b are roots of $x^2 - mx + 2 = 0$, we have

$$x^2 - mx + 2 = (x - a)(x - b) \quad \text{and} \quad ab = 2.$$

In a similar manner, the constant term of $x^2 - px + q$ is the product of $a + (1/b)$ and $b + (1/a)$, so

$$q = \left(a + \frac{1}{b}\right)\left(b + \frac{1}{a}\right) = ab + 1 + 1 + \frac{1}{ab} = \frac{9}{2}.$$

2002A

14. **(B)** Let p and q be two primes that are roots of $x^2 - 63x + k = 0$. Then

$$x^2 - 63x + k = (x - p)(x - q) = x^2 - (p + q)x + p \cdot q,$$

so $p + q = 63$ and $p \cdot q = k$. Since 63 is odd, one of the primes must be 2 and the other 61. Thus, there is exactly one possible value for k, namely $k = p \cdot q = 2 \cdot 61 = 122$.

2015B

14. **Answer (D):** If $(x-a)(x-b)+(x-b)(x-c) = 0$, then $(x-b)(2x-(a+c)) = 0$, so the two roots are b and $\frac{a+c}{2}$. The maximum value of their sum is $9 + \frac{8+7}{2} = 16.5$.