4

TRIANG

2009B

12. **Answer (A):** The base of the triangle can be 1, 2, or 3, and its altitude is the distance between the two parallel lines, so there are three possible values for the area.

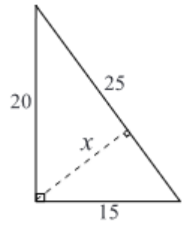
2018B

12. **Answer (C):** Let O be the center of the circle. Triangle ABC is a right triangle, and O is the midpoint of the hypotenuse $\overline{AB}$. Then

$\overline{OC}$ is a radius, and it is also one of the medians of the triangle. The centroid is located one third of the way along the median from O to C, so the centroid traces out a circle with center O and radius $\frac{1}{3} \cdot 12 = 4$ (except for the two missing points corresponding to $C = A$ or $C = B$). The area of this smaller circle is then $\pi \cdot 4^2 = 16\pi \approx 16 \cdot 3.14 \approx 50$.

2002A 13. **(B)** First notice that this is a right triangle, so two of the altitudes are the legs, whose lengths are 15 and 20. The third altitude, whose length is x, is the one drawn to the hypotenuse. The area of the triangle is $\frac{1}{2}(15)(20) = 150$. Using 25 as the base and x as the altitude, we have

$$\frac{1}{2}(25)x = 150, \quad \text{so} \quad x = \frac{300}{25} = 12.$$



OR

Since the three right triangles in the figure are similar,

$$\frac{x}{15} = \frac{20}{25}, \quad \text{so} \quad x = \frac{300}{25} = 12.$$

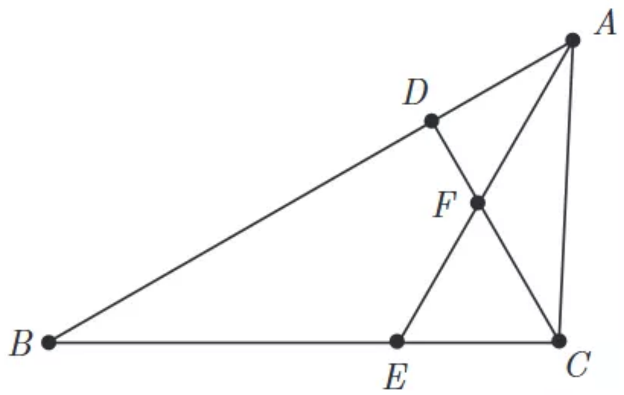
2010A 14. **Answer (C):** Let $\alpha = \angle BAE = \angle ACD = \angle ACF$. Because $\triangle CFE$ is equilateral, it follows that $\angle CFA = 120°$ and then

$$\angle FAC = 180° - 120° - \angle ACF = 60° - \alpha.$$

Therefore

$$\angle BAC = \angle BAE + \angle FAC = \alpha + (60° - \alpha) = 60°.$$

Because $AB = 2 \cdot AC$, it follows that $\triangle BAC$ is a $30-60-90°$ triangle, and thus $\angle ACB = 90°$.



2014A

14. **Answer (D):** Let the y-intercepts of lines PA and QA be $\pm b$. Then their slopes are $\frac{8\pm b}{6}$. Setting the product of the slopes to -1 and solving yields $b = \pm 10$. Therefore $\triangle APQ$ has base 20 and altitude 6, for an area of 60.

2008B

15. **Answer (A):** By the Pythagorean Theorem we have $a^2 + b^2 = (b+1)^2$, so

$$a^2 = (b+1)^2 - b^2 = 2b + 1.$$

Because b is an integer with $b < 100$, a^2 is an odd perfect square between 1 and 201, and there are six of these, namely, 9, 25, 49, 81, 121, and 169. Hence a must be 3, 5, 7, 9, 11, or 13, and there are 6 triangles that satisfy the given conditions.

2013A

15. **Answer (D):** Denote the length of the third side as x, and the altitudes to the sides of lengths 10 and 15 as m and n, respectively. Then twice the area of the triangle is $10m = 15n = \frac{1}{2}x(m+n)$. This implies that $m = \frac{3}{2}n$, so

$$15n = \frac{1}{2}x\left(\frac{3}{2}n + n\right) = \frac{5}{4}xn\,.$$

Therefore $15 = \frac{5}{4}x$, and $x = 12$.