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SPPED TIME DISTANCE WORD PROBLEMS

2008A

11. **Answer (D):** At the rate of 4 miles per hour, Steve can row a mile in 15 minutes. During that time $15 \cdot 10 = 150$ gallons of water will enter the boat. LeRoy must bail $150 - 30 = 120$ gallons of water during that time. So he must bail at the rate of at least $\frac{120}{15} = 8$ gallons per minute.

OR

Steve must row for 15 minutes to reach the shore, so the amount of water in the boat can increase by at most $\frac{30}{15} = 2$ gallons per minute. Therefore LeRoy must bail out at least $10 - 2 = 8$ gallons per minute.

2002A

12. **(B)** Let t be the number of hours Mr. Bird must travel to arrive on time. Since three minutes is the same as 0.05 hours, $40(t+0.05) = 60(t-0.05)$. Thus,

$$40t + 2 = 60t - 3, \quad \text{so } t = 0.25.$$

The distance from his home to work is $40(0.25 + 0.05) = 12$ miles. Therefore, his average speed should be $12/0.25 = 48$ miles per hour.

OR

Let d be the distance from Mr. Bird's house to work, and let s be the desired average speed. Then the desired driving time is d/s. Since $d/60$ is three minutes too short and $d/40$ is three minutes too long, the desired time must be the average, so

$$\frac{d}{s} = \frac{1}{2}\left(\frac{d}{60} + \frac{d}{40}\right).$$

This implies that $s = 48$.

2011B

12. **Answer (A):** The only parts of the track that are longer walking on the outside edge rather than the inside edge are the two semicircular portions. If the radius of the inner semicircle is r, then the difference in the lengths of the two paths is $2\pi(r+6) - 2\pi r = 12\pi$. Let x be her speed in meters per second. Then $36x = 12\pi$, and $x = \frac{\pi}{3}$.

2008B

12. **Answer (A):** During the year Pete takes

$$44 \times 10^5 + 5 \times 10^4 = 44.5 \times 10^5$$

steps. At 1800 steps per mile, the number of miles Pete walks is

$$\frac{44.5 \times 10^5}{18 \times 10^2} = \frac{44.5}{18} \times 10^3 \approx 2.5 \times 10^3 = 2500.$$

2012B

13. **Answer (C):** If the numbers are arranged in the order a, b, c, d, e, then the iterative average is

$$\frac{\frac{\frac{\frac{a+b}{2}+c}{2}+d}{2}+e}{2}=\frac{a+b+2c+4d+8e}{16}.$$

The largest value is obtained by letting $(a,b,c,d,e)=(1,2,3,4,5)$ or $(2,1,3,4,5)$, and the smallest value is obtained by letting $(a,b,c,d,e)=(5,4,3,2,1)$ or $(4,5,3,2,1)$. In the former case the iterative average is $65/16$, and in the latter case the iterative average is $31/16$, so the desired difference is

$$\frac{65}{16}-\frac{31}{16}=\frac{34}{16}=\frac{17}{8}.$$

2008A

13. **Answer (D):** In one hour Doug can paint $\frac{1}{5}$ of the room, and Dave can paint $\frac{1}{7}$ of the room. Working together, they can paint $\frac{1}{5}+\frac{1}{7}$ of the room in one hour. It takes them t hours to do the job, but because they take an hour for lunch, they work for only $t-1$ hours. The fraction of the room that they paint in this time is

$$\left(\frac{1}{5}+\frac{1}{7}\right)(t-1),$$

which must be equal to 1. It may be checked that the solution, $t=\frac{47}{12}$, does not satisfy the equation in any of the other answer choices.

2010A

13. **Answer (A):** Angelina drove $80t$ km before she stopped. After her stop, she drove $(3-\frac{1}{3}-t)$ hours at an average rate of 100 kph, so she covered $100(\frac{8}{3}-t)$ km in that time. Therefore $80t+100(\frac{8}{3}-t)=250$. Note that $t=\frac{5}{6}$.s

2006A

15. **(D)** Since Odell's rate is $5/6$ that of Kershaw, but Kershaw's lap distance is $6/5$ that of Odell, they each run a lap in the same time. Hence they pass twice each time they circle the track. Odell runs

$$(30 \text{ min})\left(250\frac{\text{m}}{\text{min}}\right)\left(\frac{1}{100\pi}\frac{\text{laps}}{\text{m}}\right) = \frac{75}{\pi} \text{ laps} \approx 23.87 \text{ laps},$$

as does Kershaw. Because $23.5 < 23.87 < 24$, they pass each other $2(23.5) = 47$ times.

2008A

15. **Answer (D):** Suppose that Ian drove for t hours at an average speed of r miles per hour. Then he covered a distance of rt miles. The number of miles Han covered by driving 5 miles per hour faster for 1 additional hour is

$$(r + 5)(t + 1) = rt + 5t + r + 5.$$

Since Han drove 70 miles more than Ian,

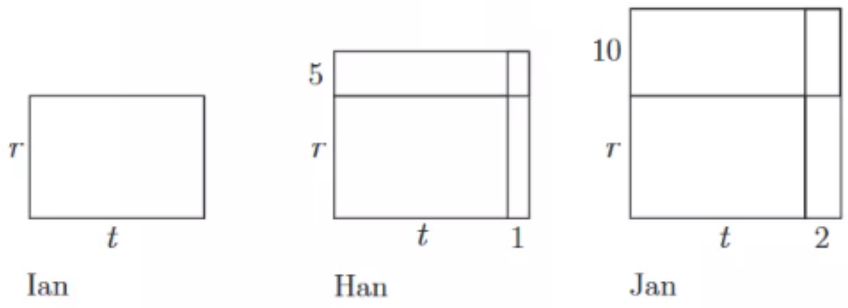
$$70 = (r + 5)(t + 1) - rt = 5t + r + 5, \quad \text{so} \quad 5t + r = 65.$$

The number of miles Jan drove more than Ian is consequently

$$(r + 10)(t + 2) - rt = 10t + 2r + 20 = 2(5t + r) + 20 = 2 \cdot 65 + 20 = 150.$$

OR

Represent the time traveled, average speed, and distance for Ian as length, width, and area, respectively, of a rectangle as shown. A similarly formed rectangle for Han would include an additional 1 unit of length and 5 units of width as compared to Ian's rectangle. Jan's rectangle would have an additional 2 units of length and 10 units of width as compared to Ian's rectangle.



Given that Han's distance exceeds that of Ian by 70 miles, and Jan's $10 \times t$ and $2 \times r$ rectangles are twice the size of Ian's $5 \times t$ and $1 \times r$ rectangles, respectively, it follows that Jan's distance exceeds that of Ian by

$$2(70 - 5) + 20 = 150 \text{ miles}.$$

2011A 15. **Answer (C):** Let x be the number of miles driven exclusively on gasoline. Then the total number of miles traveled is $x+40$, and the amount of gas used is $0.02x$ gallons. Therefore the average number of miles per gallon is

$$\frac{x+40}{0.02x}=55\,.$$

Solving results in $x=400$, so the total number of miles traveled is 440.

2014A 15. **Answer (C):** Let d be the remaining distance after one hour of driving, and let t be the remaining time until his flight. Then $d=35(t+1)$, and $d=50(t-0.5)$. Solving gives $t=4$ and $d=175$. The total distance from home to the airport is $175+35=210$ miles.

OR

Let d be the distance between David's home and the airport. The time required to drive the entire distance at 35 MPH is $\frac{d}{35}$ hours. The time required to drive at 35 MPH for the first 35 miles and 50 MPH for the remaining $d-35$ miles is $1+\frac{d-35}{50}$. The second trip is 1.5 hours quicker than the first, so

$$\frac{d}{35}-\left(1+\frac{d-35}{50}\right)=1.5.$$

Solving yields $d=210$ miles.