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STATS MEAN MEDIAN MODE

2012A

13. **Answer (C):** If the numbers are arranged in the order a, b, c, d, e, then the iterative average is

$$\frac{\frac{\frac{a+b}{2}+c}{2}+d}{2}+e}{2} = \frac{a+b+2c+4d+8e}{16}.$$

The largest value is obtained by letting $(a,b,c,d,e) = (1,2,3,4,5)$ or $(2,1,3,4,5)$, and the smallest value is obtained by letting $(a,b,c,d,e) = (5,4,3,2,1)$ or $(4,5,3,2,1)$. In the former case the iterative average is $65/16$, and in the latter case the iterative average is $31/16$, so the desired difference is

$$\frac{65}{16} - \frac{31}{16} = \frac{34}{16} = \frac{17}{8}.$$

2000

14. **Answer (C):** Note that the integer average condition means that the sum of the scores of the first n students is a multiple of n. The scores of the first two students must be both even or both odd, and the sum of the scores of the first three students must be divisible by 3. The remainders when 71, 76, 80, 82, and 91 are divided by 3 are 2, 1, 2, 1, and 1, respectively. Thus the only sum of three scores divisible by 3 is $76 + 82 + 91 = 249$, so the first two scores entered are 76 and 82 (in some order), and the third score is 91. Since 249 is 1 larger than a multiple of 4, the fourth score must be 3 larger than a multiple of 4, and the only possible is 71, leaving 80 as the score of the fifth student.

2005A

14. **(E)** The first and last digits must be both odd or both even for their average to be an integer. There are $5 \cdot 5 = 25$ odd-odd combinations for the first and last digits. There are $4 \cdot 5 = 20$ even-even combinations that do not use zero as the first digit. Hence the total is 45.

2010B

14. **Answer (B):** The average of the numbers is

$$\frac{1 + 2 + \cdots + 99 + x}{100} = \frac{\frac{99 \cdot 100}{2} + x}{100} = \frac{99 \cdot 50 + x}{100} = 100x.$$

This equation is equivalent to $9999x = (99 \cdot 101)x = 99 \cdot 50$, so $x = \frac{50}{101}$.

2018B

14. **Answer (D):** The list has $2018 - 10 = 2008$ entries that are not equal to the mode. Because the mode is unique, each of these 2008 entries can occur at most 9 times. There must be at least $\lceil \frac{2008}{9} \rceil = 224$ distinct values in the list that are different from the mode, because if there were fewer than this many such values, then the size of the list would be at most $9 \cdot (\lceil \frac{2008}{9} \rceil - 1) + 10 = 2017 < 2018$. (The ceiling function notation $\lceil x \rceil$ represents the least integer greater than or equal to x.) Therefore the least possible number of distinct values that can occur in the list is 225. One list satisfying the conditions of the problem contains 9 instances of each of the numbers 1 through 223, 10 instances of the number 224, and one instance of 225.