14

CALCULATION

2003A

16. (C) Powers of 13 have the same units digit as the corresponding powers of 3; and

$$3^1 = 3, \quad 3^2 = 9, \quad 3^3 = 27, \quad 3^4 = 81, \quad \text{and} \quad 3^5 = 243.$$

Since the units digit of 3^1 is the same as the units digit of 3^5, units digits of powers of 3 cycle through $3, 9, 7$, and 1. Hence the units digit of 3^{2000} is 1, so the units digit of 3^{2003} is 7. The same is true of the units digit of 13^{2003}.

2011A

16. **Answer (B):** Let $k = \sqrt{9-6\sqrt{2}} + \sqrt{9+6\sqrt{2}}$. Squaring both sides and simplifying results in

$$\begin{aligned} k^2 &= 9-6\sqrt{2}+2\sqrt{(9-6\sqrt{2})(9+6\sqrt{2})}+9+6\sqrt{2} \\ &= 18+2\sqrt{81-72} \\ &= 18+2\sqrt{9} \\ &= 24 \end{aligned}$$

Because $k > 0$, $k = 2\sqrt{6}$.

2013B

20. **Answer (B):** The prime factorization of 2013 is $3 \cdot 11 \cdot 61$. There must be a factor of 61 in the numerator, so $a_1 \geq 61$. Since $a_1!$ will have a factor of 59 and 2013 does not, there must be a factor of 59 in the denominator, and $b_1 \geq 59$. Thus $a_1 + b_1 \geq 120$, and this minimum value can be achieved only if $a_1 = 61$ and $b_1 = 59$. Furthermore, this minimum value is attainable because

$$2013 = \frac{(61!)(11!)(3!)}{(59!)(10!)(5!)}.$$

Thus $|a_1 - b_1| = a_1 - b_1 = 61 - 59 = 2$.

2014B

17. **Answer (D):** Note that

$$\begin{aligned} 10^{1002} - 4^{501} &= 2^{1002} \cdot 5^{1002} - 2^{1002} \\ &= 2^{1002}(5^{1002} - 1) \\ &= 2^{1002}(5^{501} - 1)(5^{501} + 1) \\ &= 2^{1002}(5-1)(5^{500} + 5^{499} + \cdots + 5 + 1)(5+1)(5^{500} - 5^{499} + \cdots \\ &\quad - 5 + 1) \\ &= 2^{1005}(3)(5^{500} + 5^{499} + \cdots + 5 + 1)(5^{500} - 5^{499} + \cdots - 5 + 1). \end{aligned}$$

Because each of the last two factors is a sum of an odd number of odd terms, they are both odd. The greatest power of 2 is 2^{1005}.