2

3D GEOMETRY

2016B 17. **Answer (D):** Suppose that one pair of opposite faces of the cube are assigned the numbers a and b, a second pair of opposite faces are assigned the numbers c and d, and the remaining pair of opposite faces are assigned the numbers e and f. Then the needed sum of products is $ace + acf + ade + adf + bce + bcf + bde + bdf = (a + b)(c + d)(e + f)$. The sum of these three factors is $2 + 3 + 4 + 5 + 6 + 7 = 27$. A product of positive numbers whose sum is fixed is maximized when the factors are all equal. Thus the greatest possible value occurs when $a + b = c + d = e + f = 9$, as in $(a, b, c, d, e, f) = (2, 7, 3, 6, 4, 5)$. This results in the value $9^3 = 729$.

2010A 17. **Answer (A):** The volume of the solid cube is 27 in^3. The first hole to be cut removes $2 \times 2 \times 3 = 12$ in^3 from the volume. The other holes remove $2 \times 2 \times 0.5 = 2$ in^3 from each of the four remaining faces. The volume of the remaining solid is $27 - 12 - 4(2) = 7$ in^3.

2002A

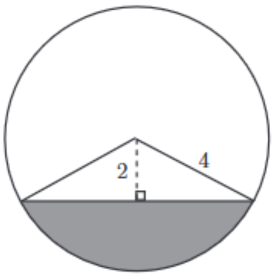
18. **(D)** There are six dice that have a single face on the surface, and these dice can be oriented so that the face with the 1 is showing. They will contribute $6(1) = 6$ to the sum. There are twelve dice that have just two faces on the surface because they are along an edge but not at a vertex of the large cube. These dice can be oriented so that the 1 and 2 are showing, and they will contribute $12(1+2) = 36$ to the sum. There are eight dice that have three faces on the surface because they are at the vertices of the large cube, and these dice can be oriented so that the 1, 2, and 3 are showing. They will contribute $8(1+2+3) = 48$ to the sum. Consequently, the minimum sum of all the numbers showing on the large cube is $6 + 36 + 48 = 90$.

2004A

19. **(C)** If the stripe were cut from the silo and spread flat, it would form a parallelogram 3 feet wide and 80 feet high. So the area of the stripe is $3(80) = 240$ square feet.

2008B

19. **Answer (E):** The portion of each end of the tank that is under water is a circular sector with two right triangles removed as shown. The hypotenuse of each triangle is 4, and the vertical leg is 2, so each is a $30-60-90^\circ$ triangle. Therefore the sector has a central angle of 120°, and the area of the sector is



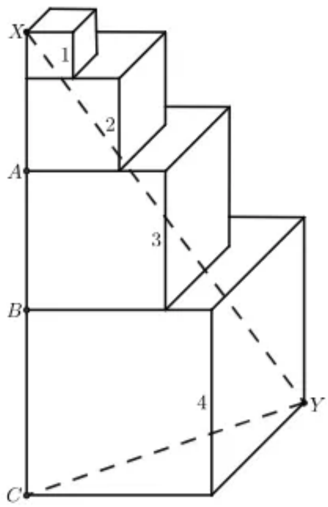
$$\frac{120}{360} \cdot \pi(4)^2 = \frac{16}{3}\pi.$$

The area of each triangle is $\frac{1}{2}(2)\left(2\sqrt{3}\right)$, so the portion of each end that is underwater has area $\frac{16}{3}\pi - 4\sqrt{3}$. The length of the cylinder is 9, so the volume of the water is

$$9\left(\frac{16}{3}\pi - 4\sqrt{3}\right) = 48\pi - 36\sqrt{3}.$$

2014A

19. **Answer (A):** Label vertices A, B, and C as shown. Note that $XC = 10$ and $CY = \sqrt{4^2+4^2} = 4\sqrt{2}$. Because $\triangle XYC$ is a right triangle, $XY = \sqrt{10^2+(4\sqrt{2})^2} = 2\sqrt{33}$. The ratio of BX to CX is $\frac{3}{5}$, so in the top face of the bottom cube the distance from B to $\overline{XY}$ is $4\sqrt{2}\cdot\frac{3}{5} = \frac{12\sqrt{2}}{5}$. This distance is less than $3\sqrt{2}$, so $\overline{XY}$ pierces the top and bottom faces of the cube with side length 3. The ratio of AB to XC is $\frac{3}{10}$, so the length of $\overline{XY}$ that is inside the cube with side length 3 is $\frac{3}{10}\cdot 2\sqrt{33} = \frac{3\sqrt{33}}{5}$.



OR

Place the figure in a 3-dimensional coordinate system with the lower left front corner at $(0,0,0)$, $X = (0,0,10)$, and $Y = (4,4,0)$. Then line XY consists of all points of the form $(4t, 4t, 10-10t)$. This line intersects the bottom face of the cube with side length 3 when $10-10t = 4$, or $t = \frac{3}{5}$; this is the point $(\frac{12}{5}, \frac{12}{5}, 4)$, and because $\frac{12}{5} < 3$, the point indeed lies on that face. Similarly, line XY intersects the top face of the cube with side length 3 when $10-10t = 7$, or $t = \frac{3}{10}$; this is the point $(\frac{6}{5}, \frac{6}{5}, 7)$. Therefore the desired length is

$$\sqrt{\left(\frac{12}{5}-\frac{6}{5}\right)^2+\left(\frac{12}{5}-\frac{6}{5}\right)^2+(4-7)^2} = \frac{3}{5}\sqrt{33}.$$