8

BIRTHDAYS

2004B 17. **(B)** Let Jack's age be $10x + y$ and Bill's age be $10y + x$. In five years Jack will be twice as old as Bill. Therefore

$$10x + y + 5 = 2(10y + x + 5),$$

so $8x = 19y + 5$. The expression $19y + 5 = 16y + 8 + 3(y - 1)$ is a multiple of 8 if and only if $y - 1$ is a multiple of 8. Since both x and y are 9 or less, the only solution is $y = 1$ and $x = 3$. Thus Jack is 31 and Bill is 13, so the difference between their ages is 18.

2018B

19. **Answer (E):** Let Chloe be n years old today, so she is $n-1$ years older than Zoe. For integers $y \geq 0$, Chloe's age will be a multiple of Zoe's age y years from now if and only if

$$\frac{n+y}{1+y} = 1 + \frac{n-1}{1+y}$$

is an integer, that is, $1+y$ is a divisor of $n-1$. Thus $n-1$ has exactly 9 positive integer divisors, so the prime factorization of $n-1$ has one of the two forms p^2q^2 or p^8. There are no two-digit integers of the form p^8, and the only one of the form p^2q^2 is $2^2 \cdot 3^2 = 36$. Therefore Chloe is 37 years old today, and Joey is 38. His age will be a multiple of Zoe's age in y years if and only if $1+y$ is a divisor of $38-1=37$. The nonnegative integer solutions for y are 0 and 36, so the only other time Joey's age will be a multiple of Zoe's age will be when he is $38+36=74$ years old. The requested sum is $7+4=11$.