13

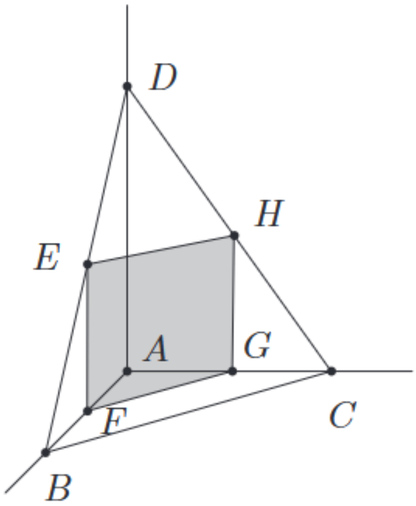
GEOMETERY WORD PROBLEMS

2012B

21. **Answer (C):** The midpoint formula gives $E = (\frac{1}{2}, 0, \frac{3}{2})$, $F = (\frac{1}{2}, 0, 0)$, $G = (0, 1, 0)$, and $H = (0, 1, \frac{3}{2})$. Note that $EF = GH = \frac{3}{2}$, $\overline{EF} \perp \overline{EH}$, $\overline{GF} \perp \overline{GH}$, and

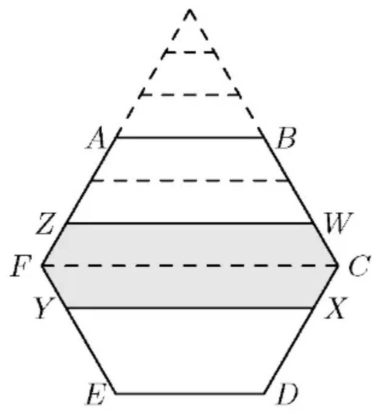
$$EH = FG = \sqrt{\left(\frac{1}{2}\right)^2 + 1^2} = \frac{\sqrt{5}}{2}.$$

Therefore $EFGH$ is a rectangle with area $\frac{3}{2} \cdot \frac{\sqrt{5}}{2} = \frac{3\sqrt{5}}{4}$.



2016B

23. **Answer (C):** Extend sides $\overline{CB}$ and $\overline{FA}$ to meet at G. Note that $FC = 2AB$ and $ZW = \frac{5}{3}AB$. Then the areas of $\triangle BAG$, $\triangle WZG$, and $\triangle CFG$ are in the ratio $1^2 : (\frac{5}{3})^2 : 2^2 = 9 : 25 : 36$. Thus $\frac{[ZWCF]}{[ABCF]} = \frac{36-25}{36-9} = \frac{11}{27}$, and by symmetry, $\frac{[WCXYFZ]}{[ABCDEF]} = \frac{11}{27}$ also.



OR

Suppose that $AB = 1$; then $FZ = \frac{1}{3}$ and $FC = 2$. Trapezoid $WCFZ$, which is the upper half of hexagon $WCXYFZ$, can be tiled by 11 equilateral triangles of side length $\frac{1}{3}$, and the lower half similarly, making 22 such triangles. Hexagon $ABCDEF$ can be tiled by 6 equilateral triangles of side length 1, and each of these can be tiled by 9 equilateral triangles of side length $\frac{1}{3}$, making a total of $6 \cdot 9 = 54$ small triangles. The required ratio is $\frac{22}{54} = \frac{11}{27}$.

2005A

25. **(D)** We have

$$\frac{\text{Area}(ADE)}{\text{Area}(ABE)} = \frac{AD}{AB} = \frac{19}{25} \quad \text{and} \quad \frac{\text{Area}(ABE)}{\text{Area}(ABC)} = \frac{AE}{AC} = \frac{14}{42} = \frac{1}{3},$$

so

$$\frac{\text{Area}(ABC)}{\text{Area}(ADE)} = \frac{25}{19} \cdot \frac{3}{1} = \frac{75}{19},$$

and

$$\frac{\text{Area}(BCED)}{\text{Area}(ADE)} = \frac{\text{Area}(ABC) - \text{Area}(ADE)}{\text{Area}(ADE)} = \frac{75}{19} - 1 = \frac{56}{19}.$$

Thus

$$\frac{\text{Area}(ADE)}{\text{Area}(BCED)} = \frac{19}{56}.$$