5

SPEED TIME DISTANCE

2002B   21. **(B)** Let A be the number of square feet in Andy's lawn. Then $A/2$ and $A/3$ are the areas of Beth's lawn and Carlos' lawn, respectively, in square feet. Let R be the rate, in square feet per minute, that Carlos' lawn mower cuts. Then Beth's mower and Andy's mower cut at rates of $2R$ and $3R$ square feet per minute, respectively. Thus,

$$\text{Andy takes } \frac{A}{3R} \text{ minutes to mow his lawn,}$$

$$\text{Beth takes } \frac{A/2}{2R} = \frac{A}{4R} \text{ minutes to mow hers,}$$
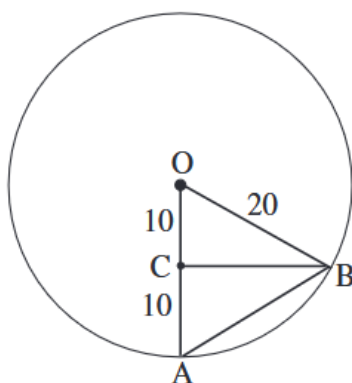
and

$$\text{Carlos takes } \frac{A/3}{R} = \frac{A}{3R} \text{ minutes to mow his.}$$

Since $\dfrac{A}{4R} < \dfrac{A}{3R}$, Beth will finish first.

2002B   24. **(D)** In the figure, the center of the wheel is at O, and the rider travels from A to B. Since $AC = 10$ and $OB = OA = 20$, the point C is the midpoint of $\overline{OA}$. In the right $\triangle OCB$, we have OC half of the length of the hypotenuse OB, so $m\angle COB = 60°$. Since the wheel turns through an angle of $360°$ in 60 seconds, the time required to turn through an angle of $60°$ is
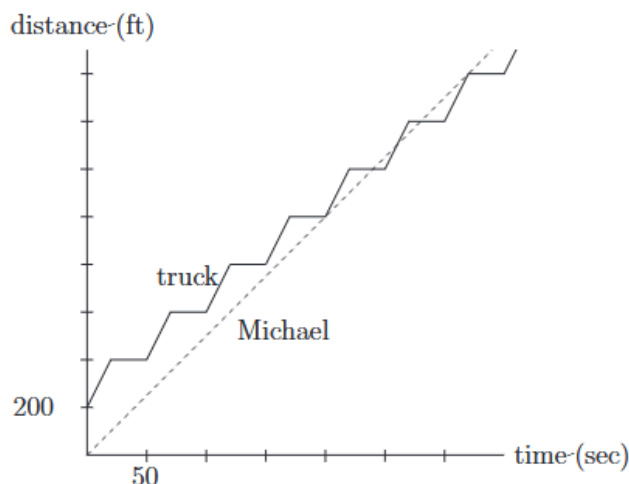
$$60 \left(\frac{60}{360} \right) = 10 \text{ seconds.}$$

2008B

25. **Answer (B):** Number the pails consecutively so that Michael is presently at pail 0 and the garbage truck is at pail 1. Michael takes $200/5 = 40$ seconds to walk between pails, so for $n \geq 0$ he passes pail n after $40n$ seconds. The truck takes 20 seconds to travel between pails and stops for 30 seconds at each pail. Thus for $n \geq 1$ it leaves pail n after $50(n-1)$ seconds, and for $n \geq 2$ it arrives at pail n after $50(n-1) - 30$ seconds. Michael will meet the truck at pail n if and only if

$$50(n-1) - 30 \leq 40n \leq 50(n-1) \quad \text{or, equivalently,} \quad 5 \leq n \leq 8.$$

Hence Michael first meets the truck at pail 5 after 200 seconds, just as the truck leaves the pail. He passes the truck at pail 6 after 240 seconds and at pail 7 after 280 seconds. Finally, Michael meets the truck just as it arrives at pail 8 after 320 seconds. These conditions imply that the truck is ahead of Michael between pails 5 and 6 and that Michael is ahead of the truck between pails 7 and 8. However, the truck must pass Michael at some point between pails 6 and 7, so they meet a total of five times.