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STATS MEAN MEDIAN MODE

2002A

21. **(D)** The values 6, 6, 6, 8, 8, 8, 8, 14 satisfy the requirements of the problem, so the answer is at least 14. If the largest number were 15, the collection would have the ordered form 7, __, __, 8, 8, __, __, 15. But $7 + 8 + 8 + 15 = 38$, and a mean of 8 implies that the sum of all values is 64. In this case, the four missing values would sum to $64 - 38 = 26$, and their average value would be 6.5. This implies that at least one would be less than 7, which is a contradiction. Therefore, the largest integer that can be in the set is 14.

2000 23. **Answer (E):** If x were less than or equal to 2, then 2 would be both the median and the mode of the list. Thus $x > 2$. Consider the two cases $2 < x < 4$, and $x \geq 4$.

Case 1: If $2 < x < 4$, then 2 is the mode, x is the median, and $\frac{25+x}{7}$ is the mean, which must equal $2-(x-2)$, $\frac{x+2}{2}$, or $x+(x-2)$, depending on the size of the mean relative to 2 and x. These give $x = \frac{3}{8}$, $x = \frac{36}{5}$, and $x = 3$, of which $x = 3$ is the only value between 2 and 4.

Case 2: If $x \geq 4$, then 4 is the median, 2 is the mode, and $\frac{25+x}{7}$ is the mean, which must be 0,3, or 6. Thus $x = -25$, -4, or 17, of which 17 is the only one of these values greater than or equal to 4.

Thus the x-value sum to $3 + 17 = 20$.

2002B 25. **(A)** Let n denote the number of integers in the original list, and m the original mean. Then the sum of the original numbers is mn. After 15 is appended to the list, we have the sum

$$(m+2)(n+1) = mn + 15, \quad \text{so} \quad m + 2n = 13.$$

After 1 is appended to the enlarged list, we have the sum

$$(m+1)(n+2) = mn + 16, \quad \text{so} \quad 2m + n = 14.$$

Solving $m + 2n = 13$ and $2m + n = 14$ gives $m = 5$ and $n = 4$.

2017B 25. **Answer (E):** Let S be the sum of Isabella's 7 scores. Then S is a multiple of 7, and

$$658 = 91 + 92 + 93 + \cdots + 97 \leq S \leq 94 + 95 + 96 + \cdots + 100 = 679,$$

so S is one of 658, 665, 672, or 679. Because $S - 95$ is a multiple of 6, it follows that $S = 665$. Thus the sum of Isabella's first 6 scores was $665 - 95 = 570$, which is a multiple of 5, and the sum of her first 5 scores was also a multiple of 5. Therefore her sixth score must have been a multiple of 5. Because her seventh score was 95 and her scores were all different, her sixth score was 100. One possible sequence of scores is $91, 93, 92, 96, 98, 100, 95$.