1

can be solved by counting, no formula 2D geometry

2003B

6. (D) The height, length, and diagonal are in the ratio 3 : 4 : 5. The length of the diagonal is 27, so the horizontal length is

$$\frac{4}{5}(27) = 21.6 \text{ inches.}$$

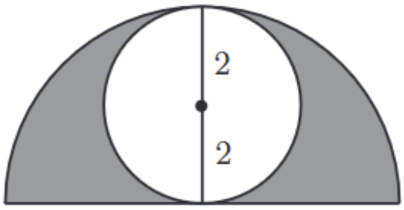
2006B

6. **(D)** Since the square has side length $2/\pi$, the diameter of each circular section is $2/\pi$. The boundary of the region consists of 4 semicircles, whose total perimeter is twice the circumference of a circle having diameter $2/\pi$. Hence the perimeter of the region is

$$2 \cdot \left(\pi \cdot \frac{2}{\pi}\right) = 4.$$

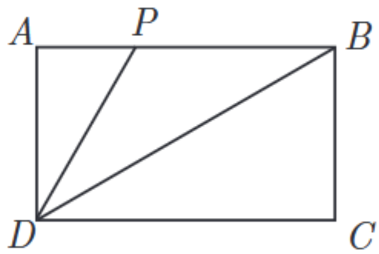
2009A

6. **Answer (A):** The semicircle has radius 4 and total area $\frac{1}{2} \cdot \pi \cdot 4^2 = 8\pi$. The area of the circle is $\pi \cdot 2^2 = 4\pi$. The fraction of the area that is not shaded is $\frac{4\pi}{8\pi} = \frac{1}{2}$, and hence the fraction of the area that is shaded is also $\frac{1}{2}$.



2000

7. **Answer (B):** Both triangles APD and CBD are $30-60-90°$ triangles. Thus $DP = \frac{2\sqrt{3}}{3}$ and $DB = 2$. Since $\angle BDP = \angle PDB$, it follows that $PB = PD = \frac{2\sqrt{3}}{3}$. Hence the perimeter of $\triangle BDP$ is $\frac{2\sqrt{3}}{3} + \frac{2\sqrt{3}}{3} + 2 = 2 + \frac{4\sqrt{3}}{3}$.

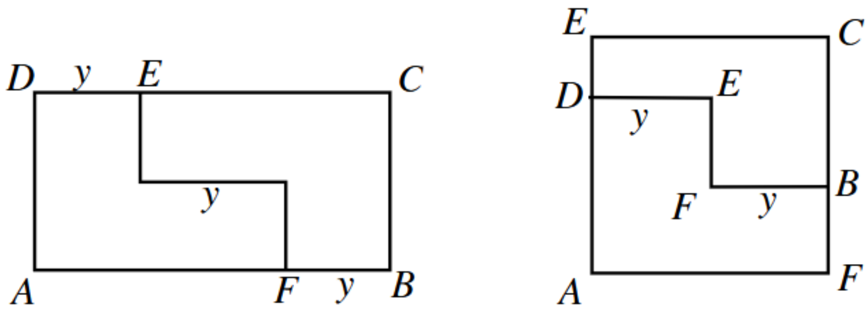


2006A

7. **(A)** Let E represent the end of the cut on $\overline{DC}$, and let F represent the end of the cut on $\overline{AB}$. For a square to be formed, we must have

$$DE = y = FB \quad \text{and} \quad DE + y + FB = 18, \quad \text{so} \quad y = 6.$$

The rectangle that is formed by this cut is indeed a square, since the original rectangle has area $8 \cdot 18 = 144$, and the rectangle that is formed by this cut has a side of length $2 \cdot 6 = 12 = \sqrt{144}$.



2018B

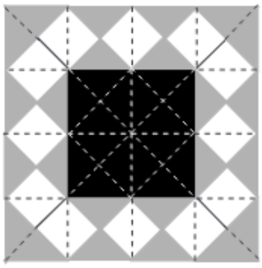
7. **Answer (D):** Suppose without loss of generality that each small semicircle has radius 1; then the large semicircle has radius N. The area of each small semicircle is $\frac{\pi}{2}$, and the area of the large semicircle is $N^2 \cdot \frac{\pi}{2}$. The combined area A of the N small semicircles is $N \cdot \frac{\pi}{2}$, and the area B inside the large semicircle but outside the small semicircles is

$$N^2 \cdot \frac{\pi}{2} - N \cdot \frac{\pi}{2} = \left(N^2 - N\right) \cdot \frac{\pi}{2}.$$

Thus the ratio $A : B$ of the areas is $N : \left(N^2 - N\right)$, which is $1 : (N-1)$. Because this ratio is given to be $1 : 18$, it follows that $N - 1 = 18$ and $N = 19$.

2002A

8. **(A)** Draw additional lines to cover the entire figure with congruent triangles. There are 24 triangles in the blue region, 24 in the white region, and 16 in the red region. Thus, $B = W$.



2005A

8. **(C)** The symmetry of the figure implies that $\triangle ABH$, $\triangle BCE$, $\triangle CDF$, and $\triangle DAG$ are congruent right triangles. So

$$BH = CE = \sqrt{BC^2 - BE^2} = \sqrt{50 - 1} = 7,$$

and $EH = BH - BE = 7 - 1 = 6$. Hence the square $EFGH$ has area $6^2 = 36$.

OR

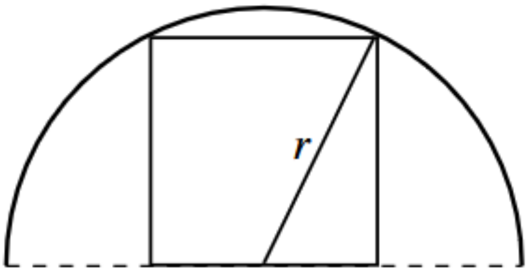
As in the first solution, $BH = 7$. Now note that $\triangle ABH$, $\triangle BCE$, $\triangle CDF$, and $\triangle DAG$ are congruent right triangles, so

$$\text{Area}(EFGH) = \text{Area}(ABCD) - 4\text{Area}(\triangle ABH) = 50 - 4\left(\tfrac{1}{2} \cdot 1 \cdot 7\right) = 36.$$

2005B 8. **(A)** The four white quarter circles in each tile have the same area as a whole circle of radius 1/2, that is, $\pi(1/2)^2 = \pi/4$ square feet. So the area of the shaded portion of each tile is $1 - \pi/4$ square feet. Since there are $8 \cdot 10 = 80$ tiles in the entire floor, the area of the total shaded region in square feet is

$$80\left(1 - \frac{\pi}{4}\right) = 80 - 20\pi.$$

2006B 8. **(B)**The square has side length $\sqrt{40}$.



Let r be the radius of the semicircle. Then

$$r^2 = \left(\sqrt{40}\right)^2 + \left(\frac{\sqrt{40}}{2}\right)^2 = 40 + 10 = 50,$$

so the area of the semicircle is $\frac{1}{2}\pi r^2 = 25\pi$.

2015B 8. **Answer (E):** The first rotation results in Figure 1, the reflection in Figure 2, and the half turn in Figure 3.

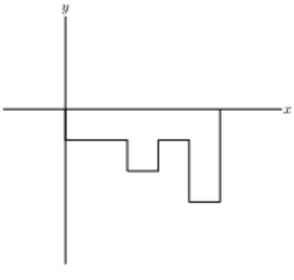


Figure 1

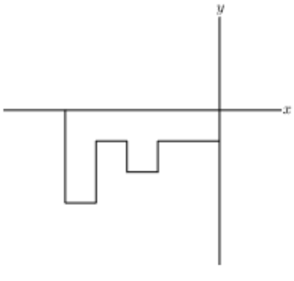


Figure 2

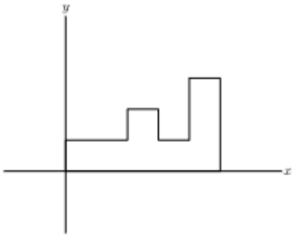


Figure 3

2018B

8. **Answer (C):** In the staircase with n steps, the number of vertical toothpicks is

$$1+2+3+\cdots+n+n=\frac{n(n+1)}{2}+n.$$

There are an equal number of horizontal toothpicks, for a total of $n(n+1)+2n$ toothpicks. Solving $n(n+1)+2n=180$ with $n>0$ yields $n=12$.

OR

By inspection, the number of toothpicks for staircases consisting of 1, 2, and 3 steps are 4, 10, and 18, respectively. The n-step staircase is obtained from the $(n-1)$-step staircase by adding $n+1$ horizontal toothpicks and $n+1$ vertical toothpicks. With this observation, the pattern can be continued so that 28, 40, 54, 70, 88, 108, 130, 154, and 180 are the numbers of toothpicks used to construct staircases consisting of 4 through 12 steps, respectively. Therefore 180 toothpicks are needed for the 12-step staircase.

2004A

9. **(B)** Let x, y, and z be the areas of $\triangle ADE$, $\triangle BDC$, and $\triangle ABD$, respectively. The area of $\triangle ABE$ is $(1/2)(4)(8)=16=x+z$, and the area of $\triangle BAC$ is $(1/2)(4)(6)=12=y+z$. The requested difference is

$$x-y=(x+z)-(y+z)=16-12=4.$$

2009B

9. **Answer (A):** Because $\triangle ABC$ is isosceles, $\angle A=\angle C$. Because $\angle A=\frac{5}{2}\angle B$, we have $\frac{5}{2}\angle B+\frac{5}{2}\angle B+\angle B=180°$, so $\angle B=30°$. Therefore $\angle ACB=\angle DCE=75°$. Because $\triangle CDE$ is isosceles, $2\angle D+75°=180°$, so $\angle D=52.5°$.

2011B

9. **Answer (D):** The area of $\triangle ABC$ is $\frac{1}{2} \cdot 3 \cdot 4 = 6$, so the area of $\triangle EBD$ is $\frac{1}{3} \cdot 6 = 2$. Note that $\triangle ABC$ and $\triangle EBD$ are right triangles with an angle in common, so they are similar. Therefore BD and DE are in the ratio 4 to 3. Let $BD = x$ and $DE = \frac{3}{4}x$. Then the area of $\triangle EBD$ can be expressed as $\frac{1}{2} \cdot x \cdot \frac{3}{4}x = \frac{3}{8}x^2$. Because $\triangle EBD$ has area 2, solving yields $BD = \frac{4\sqrt{3}}{3}$.

OR

Because $\triangle EBD$ and $\triangle ABC$ are similar triangles, their areas are in the ratio of the squares of their corresponding linear parts. Therefore $\left(\frac{BD}{4}\right)^2 = \frac{1}{3}$ and $BD = \frac{4\sqrt{3}}{3}$.

2015B

9. **Answer (B):** The shaded area is obtained by subtracting the area of the semicircle from the area of the quarter circle:

$$\frac{1}{4}\pi \cdot 3^2 - \frac{1}{2}\pi\left(\frac{3}{2}\right)^2 = \frac{9\pi}{4} - \frac{9\pi}{8} = \frac{9\pi}{8}.$$

2018A

9. **Answer (E):** The length of the base $\overline{DE}$ of $\triangle ADE$ is 4 times the length of the base of a small triangle, so the area of $\triangle ADE$ is $4^2 \cdot 1 = 16$. Therefore the area of $DBCE$ is the area of $\triangle ABC$ minus the area of $\triangle ADE$, which is $40 - 16 = 24$.

2009A

10. **Answer (B):** By the Pythagorean Theorem, $AB^2 = BD^2+9$, $BC^2 = BD^2+16$, and $AB^2 + BC^2 = 49$. Adding the first two equations and substituting gives $2 \cdot BD^2 + 25 = 49$. Then $BD = 2\sqrt{3}$, and the area of $\triangle ABC$ is $\frac{1}{2} \cdot 7 \cdot 2\sqrt{3} = 7\sqrt{3}$.

OR

Because $\triangle ADB$ and $\triangle BDC$ are similar, $\frac{BD}{3} = \frac{4}{BD}$, from which $BD = 2\sqrt{3}$. Therefore the area of $\triangle ABC$ is $\frac{1}{2} \cdot 7 \cdot 2\sqrt{3} = 7\sqrt{3}$.

2012B

10. **Answer (C):** Let a be the initial term and d the common difference for the arithmetic sequence. Then the sum of the degree measures of the central angles is

$$a + (a+d) + \cdots + (a+11d) = 12a + 66d = 360,$$

so $2a + 11d = 60$. Letting $d = 4$ yields the smallest possible positive integer value for a, namely $a = 8$.

2016A

toll. In the second crossing he must have started with 30 coins in order to have $20 + 40 = 60$ before paying the toll. So he must have started with 35 coins in order to have $30 + 40 = 70$ before paying the toll for the first crossing.

OR

Let c be the number of coins Fox had at the beginning. After three crossings he had $2(2(2c-40)-40)-40 = 8c-280$ coins. Setting this equal to 0 and solving gives $c = 35$.