11

PROBABILTY

2018B

6. **Answer (D):** Three draws will be required if and only if the first two chips drawn have a sum of 4 or less. The draws $(1,2)$, $(2,1)$, $(1,3)$, and $(3,1)$ are the only draws meeting this condition. There are $5 \cdot 4 = 20$ possible two-chip draws, so the requested probability is $\frac{4}{20} = \frac{1}{5}$. (Note that all 20 possible two-chip draws are considered in determining the denominator, even though some draws will end after the first chip is drawn.)

2003A

8. (E) The factors of 60 are

$$1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, \text{ and } 60.$$

Six of the twelve factors are less than 7, so the probability is $1/2$.

2005A

9. **(B)** There are three X's and two O's, and the tiles are selected without replacement, so the probability is

$$\frac{3}{5} \cdot \frac{2}{4} \cdot \frac{2}{3} \cdot \frac{1}{2} \cdot \frac{1}{1} = \frac{1}{10}.$$

OR

The three tiles marked X are equally likely to lie in any of $\binom{5}{3} = 10$ positions, so the probability of this arrangement is $1/10$.

2005B 9. **(D)** An odd sum requires either that the first die is even and the second is odd or that the first die is odd and the second is even. The probability is

$$\frac{1}{3} \cdot \frac{1}{3} + \frac{2}{3} \cdot \frac{2}{3} = \frac{1}{9} + \frac{4}{9} = \frac{5}{9}.$$

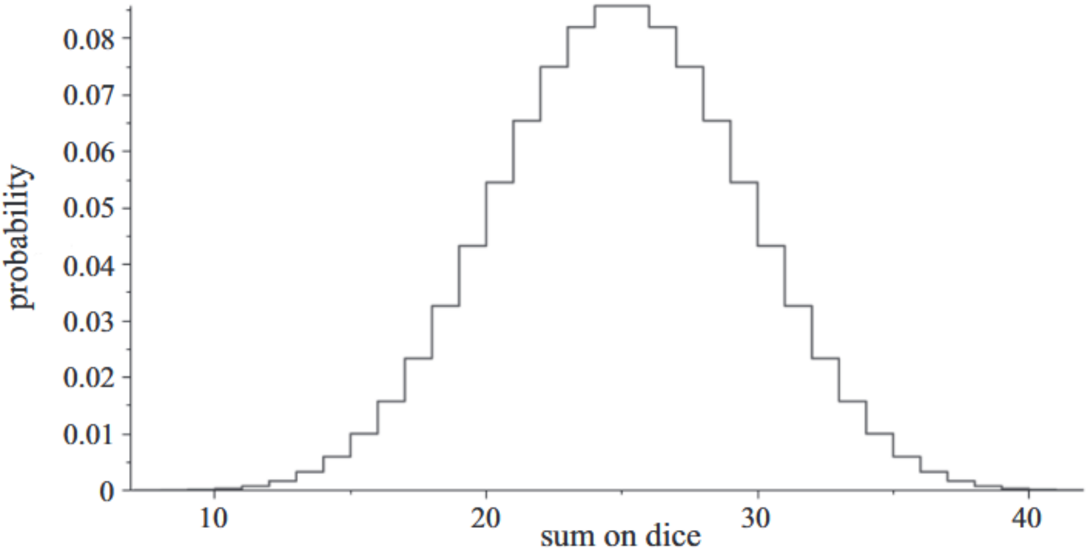
2012A 9. **Answer (D):** The sum could be 7 only if the even die showed 2 and the odd showed 5, the even showed 4 and the odd showed 3, or the even showed 6 and the odd showed 1. Each of these events can occur in $2 \cdot 2 = 4$ ways. Hence there are 12 ways for a 7 to occur. There are $6 \cdot 6 = 36$ possible outcomes, so the probability that a 7 occurs is $\frac{12}{36} = \frac{1}{3}$.

2017B

9. **Answer (D):** The probability of getting all 3 questions right is $\left(\frac{1}{3}\right)^3 = \frac{1}{27}$. Because there are 3 ways to get 2 of the questions right and 1 wrong, the probability of getting exactly 2 right is $3\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right) = \frac{6}{27}$. Therefore the probability of winning is $\frac{1}{27} + \frac{6}{27} = \frac{7}{27}$.

2018B

9. **Answer (D):** Without loss of generality, one can assume that the numbers on opposite faces of each die add up to 7. In other words, the 1 is opposite the 6, the 2 is opposite the 5, and the 3 is opposite the 4. (In fact, standard dice are numbered in this way.) The top faces give a sum of 10 if and only if the bottom faces give a sum of $7 \cdot 7 - 10 = 39$. By symmetry, the probability that the top faces give a sum of 39 is also p. The distribution of the outcomes of the dice rolls has the bell-shaped graph shown below, so no other outcome has the same probability as 10 and 39.



2004A

10. **(D)** The result will occur when both A and B have either 0, 1, 2, or 3 heads, and these probabilities are shown in the table.

Heads	0	1	2	3
A	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$
B	$\frac{1}{16}$	$\frac{4}{16}$	$\frac{6}{16}$	$\frac{4}{16}$

The probability of both coins having the same number of heads is

$$\frac{1}{8} \cdot \frac{1}{16} + \frac{3}{8} \cdot \frac{4}{16} + \frac{3}{8} \cdot \frac{6}{16} + \frac{1}{8} \cdot \frac{4}{16} = \frac{35}{128}.$$