16

SOLVE FOR X

2006A

6. **(B)** Take the seventh root of both sides to get $(7x)^2 = 14x$. Then $49x^2 = 14x$, and because $x \neq 0$ we have $49x = 14$. Thus $x = 2/7$.

2014A

7. **Answer (B):** Note that $x + y < a + y < a + b$, so inequality I is true. If $x = -2$, $y = -2$, $a = -1$, and $b = -1$, then none of the other three inequalities is true.

2007A

9. **Answer (E):** The given equations are equivalent, respectively, to

$$3^a = 3^{4(b+2)} \quad \text{and} \quad 5^{3b} = 5^{a-3}.$$

Therefore $a = 4(b + 2)$ and $3b = a - 3$. The solution of this system is $a = -12$ and $b = -5$, so $ab = 60$.

2003B 9. (B) Write all the terms with the common base 5. Then

$$5^{-4} = 25^{-2} = \frac{5^{48/x}}{5^{26/x} \cdot 25^{17/x}} = \frac{5^{48/x}}{5^{26/x} \cdot 5^{34/x}} = 5^{(48-26-34)/x} = 5^{-12/x}.$$

It follows that $-\frac{12}{x} = -4$, so $x = 3$.

OR

First write 25 as 5^2. Raising both sides to the x power gives

$$5^{-4x} = \frac{5^{48}}{5^{26}5^{34}} = 5^{48-26-34} = 5^{-12}.$$

So $-4x = -12$ and $x = 3$.

2008A 9. **Answer (B):** Because

$$\frac{2x}{3} - \frac{x}{6} = \frac{x}{2}$$

is an integer, x must be even. The case $x = 4$ shows that x is not necessarily a multiple of 3 and that none of the other statements must be true.

2001 10. **(D)** Since

$$x = \frac{24}{y} = 48z$$

we have $z = 2y$. So $72 = 2y^2$, which implies that $y = 6$, $x = 4$, and $z = 12$. Hence $x + y + z = 22$.

OR

Take the product of the equations to get $xy \cdot xz \cdot yz = 24 \cdot 48 \cdot 72$. Thus

$$(xyz)^2 = 2^3 \cdot 3 \cdot 2^4 \cdot 3 \cdot 2^3 \cdot 3^2 = 2^{10} \cdot 3^4.$$

So $(xyz)^2 = (2^5 \cdot 3^2)^2$, and we have $xyz = 2^5 \cdot 3^2$. Therefore,

$$x = \frac{xyz}{yz} = \frac{2^5 \cdot 3^2}{2^3 \cdot 3^2} = 4.$$

From this it follows that $y = 6$ and $z = 12$, so the sum is $4 + 6 + 12 = 22$.

2006A 10. **(E)** Suppose that $k = \sqrt{120 - \sqrt{x}}$ is an integer. Then $0 \le k \le \sqrt{120}$, and because k is an integer, we have $0 \le k \le 10$. Thus there are 11 possible integer values of k. For each such k, the corresponding value of x is $\left(120 - k^2\right)^2$. Because $\left(120 - k^2\right)^2$ is positive and decreasing for $0 \le k \le 10$, the 11 values of x are distinct.