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ALGEBRA WORD PROBLEMS

2012A

6. **Answer (D):** Let $x > 0$ be the first number, and let $y > 0$ be the second number. The first statement implies $xy = 9$. The second statement implies $\frac{1}{x} = \frac{4}{y}$, so $y = 4x$. Substitution yields $x \cdot (4x) = 9$, so $x = \sqrt{\frac{9}{4}} = \frac{3}{2}$. Therefore $x + y = \frac{3}{2} + 4 \cdot \frac{3}{2} = \frac{15}{2}$.

2012B

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2012B

7. **Answer (C):** The ratio of blue marbles to red marbles is $3 : 2$. If the number of red marbles is doubled, the ratio will be $3 : 4$, and the fraction of marbles that are red will be $\frac{4}{3+4} = \frac{4}{7}$.

2016A

6. **Answer (D):** Each time Emilio replaces a 2 in the ones position by 1, Ximena's sum is decreased by 1. When Emilio replaces a 2 in the tens position by 1, Ximena's sum is decreased by 10. Ximena wrote 3 twos in the ones position $(2, 12, 22)$ and 10 twos in the tens position $(20, 21, 22, \ldots, 29)$. Thus Ximena's sum is greater than Emilio's sum by $3 \cdot 1 + 10 \cdot 10 = 103$.

2001

7. **(C)** If x is the number, then moving the decimal point four places to the right is the same as multiplying x by 10,000. That is, $10{,}000x = 4 \cdot (\frac{1}{x})$, which is equivalent to $x^2 = 4/10{,}000$. Since x is positive, it follows that $x = 2/100 = 0.02$.

2004A

8. **(B)** After three rounds the players A, B, and C have 14, 13, and 12 tokens, respectively. Every subsequent three rounds of play reduces each player's supply of tokens by one. After 36 rounds they have 3, 2, and 1 token, respectively, and after the 37^{th} round Player A has no tokens.

2012A

8. **Answer (D):** Let the three whole numbers be $a < b < c$. The set of sums of pairs of these numbers is $(a+b, a+c, b+c) = (12, 17, 19)$. Thus $2(a+b+c) = (a+b)+(a+c)+(b+c) = 12+17+19 = 48$, and $a+b+c = 24$. If follows that $(a, b, c) = (24-19, 24-17, 24-12) = (5, 7, 12)$. Therefore the middle number is 7.

2010A

9. **Answer (E):** Let $x + 32$ be written in the form $CDDC$. Because x has three digits, $1000 < x + 32 < 1032$, and so $C = 1$ and $D = 0$. Hence $x = 1001 - 32 = 969$, and the sum of the digits of x is $9 + 6 + 9 = 24$.

2013B

9. **Answer (D):** Note that

$$27{,}000 = 2^3 \cdot 3^3 \cdot 5^3.$$

The only three pairwise relatively prime positive integers greater than 1 with a product of 27,000 are 8, 27, and 125. The sum of these numbers is 160.

2010B

9. **Answer (D):** The correct answer was $1-(2-(3-(4+e))) = 1-2+3-4-e = -2-e$. Larry's answer was $1-2-3-4+e = -8+e$. Therefore $-2-e = -8+e$, so $e = 3$.

2011A

10. **Answer (B):** Let C be the cost of a pencil in cents, N be the number of pencils each student bought, and S be the number of students who bought pencils. Then $C \cdot N \cdot S = 1771 = 7 \cdot 11 \cdot 23$, and $C > N > 1$. Because a majority of the students bought pencils, $30 \geq S > \frac{30}{2} = 15$. Therefore $S = 23, N = 7$, and $C = 11$.

2015B

10. **Answer (C):** There are 2014 negative integers strictly greater than -2015, and exactly half of them, or 1007, are odd. The product of an odd number of negative numbers is negative. Furthermore, because all factors are odd and some of them are multiples of 5, this product is an odd multiple of 5 and therefore has units digit 5.