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RATIO

2002A

7. **(A)** Let $C_A = 2\pi R_A$ be the circumference of circle A, let $C_B = 2\pi R_B$ be the circumference of circle B, and let L the common length of the two arcs. Then

$$\frac{45}{360}C_A = L = \frac{30}{360}C_B.$$

Therefore

$$\frac{C_A}{C_B} = \frac{2}{3} \quad \text{so} \quad \frac{2}{3} = \frac{2\pi R_A}{2\pi R_B} = \frac{R_A}{R_B}.$$

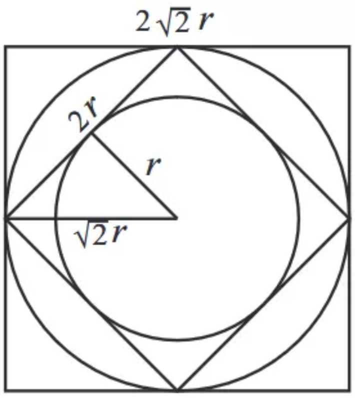
Thus, the ratio of the areas is

$$\frac{\text{Area of Circle }(A)}{\text{Area of Circle }(B)} = \frac{\pi R_A^2}{\pi R_B^2} = \left(\frac{R_A}{R_B}\right)^2 = \frac{4}{9}.$$

2005B

7. **(B)** Let the radius of the smaller circle be r. Then the side length of the smaller square is $2r$. The radius of the larger circle is half the length of the diagonal of the smaller square, so it is $\sqrt{2}r$. Hence the larger square has sides of length $2\sqrt{2}r$. The ratio of the area of the smaller circle to the area of the larger square is therefore

$$\frac{\pi r^2}{\left(2\sqrt{2}r\right)^2} = \frac{\pi}{8}.$$



2015A

6. **Answer (B):** Let x and y be the two positive numbers, with $x > y$. Then $x + y = 5(x - y)$. Thus $4x = 6y$, so $\frac{x}{y} = \frac{3}{2}$.

2016B

7. **Answer (C):** Let α and β be the measures of the angles, with $\alpha < \beta$. Then $\frac{\beta}{\alpha} = \frac{5}{4}$. Because $\alpha < \beta$, it follows that $90^\circ - \beta < 90^\circ - \alpha$, so $90^\circ - \alpha = 2(90^\circ - \beta)$. This leads to the system of linear equations $4\beta - 5\alpha = 0$ and $2\beta - \alpha = 90^\circ$. Solving the system gives $\alpha = 60^\circ$, $\beta = 75^\circ$. The requested sum is $\alpha + \beta = 135^\circ$.

2015A

8. **Answer (B):** Let p be Pete's present age, and let c be Claire's age. Then $p - 2 = 3(c - 2)$ and $p - 4 = 4(c - 4)$. Solving these equations gives $p = 20$ and $c = 8$. Thus Pete is 12 years older than Claire, so the ratio of their ages will be $2 : 1$ when Claire is 12 years old. That will occur $12 - 8 = 4$ years from now.

2011B

10. **Answer (B):** The sum of the smallest ten elements is

$$1 + 10 + 100 + \cdots + 1{,}000{,}000{,}000 = 1{,}111{,}111{,}111.$$

Hence the desired ratio is

$$\frac{10{,}000{,}000{,}000}{1{,}111{,}111{,}111} = \frac{9{,}999{,}999{,}999 + 1}{1{,}111{,}111{,}111} = 9 + \frac{1}{1{,}111{,}111{,}111} \approx 9.$$

OR

The sum of a finite geometric series of the form $a(1 + r + r^2 + \cdots + r^n)$ is $\frac{a}{1-r}(1 - r^{n+1})$. The desired denominator $1 + 10 + 10^2 + \cdots + 10^9$ is a finite geometric series with $a = 1, r = 10$, and $n = 9$. Therefore the ratio is

$$\frac{10^{10}}{1 + 10 + 10^2 + \cdots + 10^9} = \frac{10^{10}}{\frac{1}{1-10}(1 - 10^{10})} = \frac{10^{10}}{10^{10} - 1} \cdot 9 \approx \frac{10^{10}}{10^{10}} \cdot 9 = 9.$$