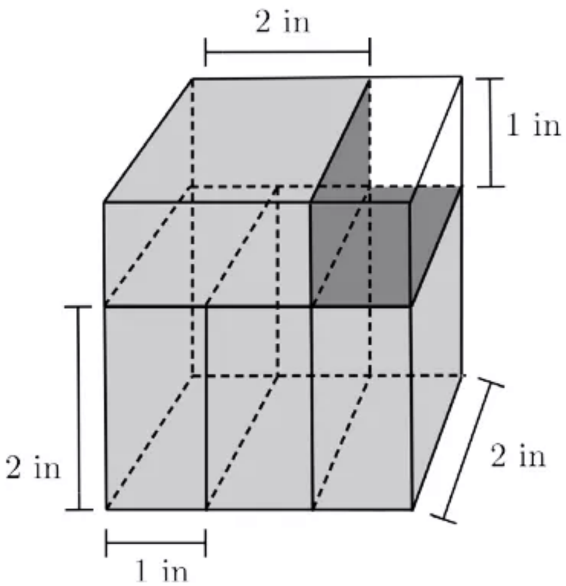
2

USES 3D GEOMETRY

2017B 6. **Answer (B):** A possible arrangement of 4 blocks is shown by the figure.



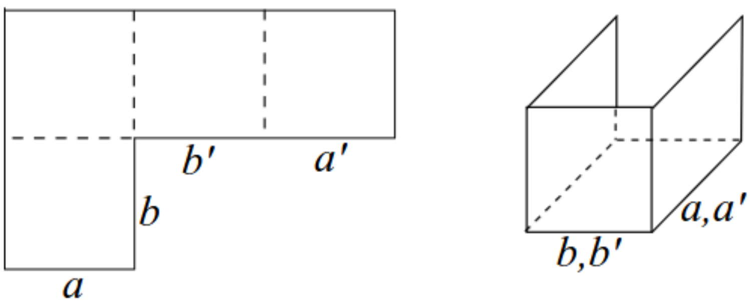
Four blocks do not completely fill the box because the combined volume of the blocks is only $4(2 \cdot 2 \cdot 1) = 16$ cubic inches, whereas the volume of the box is $3 \cdot 2 \cdot 3 = 18$ cubic inches. Because the unused space, $18 - 16 = 2$ cubic inches, is less than the volume of a block, 4 cubic inches, no more than 4 blocks can fit in the box.

2004A 7. **(C)** There are five layers in the stack, and each of the top four layers has one less orange in its length and width than the layer on which it rests. Hence the total number of oranges in the stack is

$$5 \cdot 8 + 4 \cdot 7 + 3 \cdot 6 + 2 \cdot 5 + 1 \cdot 4 = 100.$$

2015A 9. **Answer (D):** Let r, h, R, H be the radii and heights of the first and second cylinders, respectively. The volumes are equal, so $\pi r^2 h = \pi R^2 H$. Also $R = r + 0.1r = 1.1r$. Thus $\pi r^2 h = \pi(1.1r)^2 H = \pi(1.21r^2)H$. Dividing by πr^2 yields $h = 1.21H = H + 0.21H$. Thus the first height is 21% more than the second height.

2003A 10. (E) If the polygon is folded before the fifth square is attached, then edges a and a' must be joined, as must b and b'. The fifth face of the cube can be attached at any of the six remaining edges.



2018B

10. **Answer (E):** The volume of the rectangular pyramid with base $BCHE$ and apex M equals the volume of the given rectangular parallelepiped, which is 6, minus the combined volume of triangular prism $AEHDCB$, tetrahedron $BEFM$, and tetrahedron $CGHM$. Tetrahedra $BEFM$ and $CGHM$ each have three right angles at F and G, respectively, and the edges of the tetrahedra emanating from F and G have lengths 2, 3, and $\frac{1}{2}$, so the volume of each of these tetrahedra is $\frac{1}{6} \cdot (2 \cdot 3 \cdot \frac{1}{2}) = \frac{1}{2}$. The volume of the triangluar prism $AEHDCB$ is 3 because it is half the volume of the rectangular parallelepiped. Therefore the requested volume is $6 - 3 - \frac{1}{2} - \frac{1}{2} = 2$.

OR

Let P and Q be the midpoints of $\overline{BC}$ and $\overline{EH}$, respectively. By the Pythagorean Theorem $PQ = \sqrt{13}$. Let R be the foot of the perpendicular from M to $\overline{PQ}$. Then $\triangle PMQ \sim \triangle PRM$, so

$$\frac{3}{\sqrt{13}} = \frac{MQ}{PQ} = \frac{RM}{PM} = \frac{RM}{2} \quad \text{and} \quad RM = \frac{6}{\sqrt{13}}.$$

The requested volume of the pyramid is $\frac{1}{3}$ times the area of the base times the height, which is

$$\frac{1}{3} \cdot \left(\sqrt{13} \cdot 1\right) \cdot \frac{6}{\sqrt{13}} = 2.$$