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QUADRATICS

2002B

2011A

2006A

2008B 9. **Answer (A):** The quadratic formula implies that the two solutions are

$$x_1 = \frac{2a + \sqrt{4a^2 - 4ab}}{2a} \quad \text{and} \quad x_2 = \frac{2a - \sqrt{4a^2 - 4ab}}{2a},$$

so the average is

$$\frac{1}{2}(x_1 + x_2) = \frac{1}{2}\left(\frac{2a}{2a} + \frac{2a}{2a}\right) = 1.$$

OR

The sum of the solutions of a quadratic equation is the negative of the coefficient of the linear term divided by the coefficient of the quadratic term. In this case the sum of the solution is $\dfrac{-(-2a)}{a} = 2$. Hence the average of the solutions is 1 .

2002A

2002B 10. **(C)** The given conditions imply that

$$x^2 + ax + b = (x - a)(x - b) = x^2 - (a + b)x + ab,$$

so

$$a + b = -a \quad \text{and} \quad ab = b.$$

Since $b \neq 0$, the second equation implies that $a = 1$. The first equation gives $b = -2$, so $(a, b) = (1, -2)$.

2005A 10. **(A)** The quadratic formula yields

$$x = \frac{-(a+8) \pm \sqrt{(a+8)^2 - 4 \cdot 4 \cdot 9}}{2 \cdot 4}.$$

The equation has only one solution precisely when the value of the discriminant, $(a + 8)^2 - 144$, is 0. This implies that $a = -20$ or $a = 4$, and the sum is -16.

OR

The equation has one solution if and only if the polynomial is the square of a binomial with linear term $\pm\sqrt{4x^2} = \pm 2x$ and constant term $\pm\sqrt{9} = \pm 3$. Because $(2x \pm 3)^2$ has a linear term $\pm 12x$, it follows that $a + 8 = \pm 12$. Thus a is either -20 or 4, and the sum of those values is -16.

2018A