4

TRIANGLES

2008B

7. **Answer (C):** The side length of the large triangle is 10 times the side length of each small triangle, so the area of the large triangle is $10^2 = 100$ times the area of each small triangle.

2011B

7. **Answer (B):** The degree measures of two of the angles have a sum of $\frac{6}{5} \cdot 90 = 108$ and a positive difference of 30, so their measures are 69 and 39. The remaining angle has a degree measure of $180 - 108 = 72$, which is the largest angle.

2013B

7. **Answer (B):** The six points divide the circle into six arcs each measuring $60°$. By the Inscribed Angle Theorem, the angles of the triangle can only be $30°$, $60°$, $90°$, and $120°$. Because the angles of the triangle are pairwise distinct the triangle must be a $30-60-90°$ triangle. Therefore the hypotenuse of the triangle is the diameter of the circle, and the legs have lengths 1 and $\sqrt{3}$. The area of the triangle is $\frac{1}{2} \cdot 1 \cdot \sqrt{3} = \frac{\sqrt{3}}{2}$.

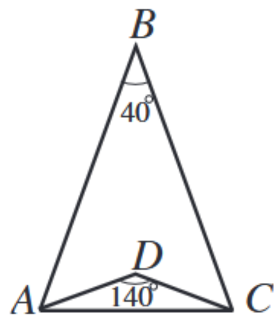
2004B

8. **(A)** Let downtown St. Paul, downtown Minneapolis, and the airport be located at S, M, and A, respectively. Then $\triangle MAS$ has a right angle at A, so by the Pythagorean Theorem,

$$MS = \sqrt{10^2 + 8^2} = \sqrt{164} \approx \sqrt{169} = 13.$$

2007A

8. **Answer (D):** Because $\triangle ABC$ is isosceles, $\angle BAC = \frac{1}{2}(180^\circ - \angle ABC) = 70^\circ$.



Similarly,

$$\angle DAC = \frac{1}{2}(180^\circ - \angle ADC) = 20^\circ.$$

Thus $\angle BAD = \angle BAC - \angle DAC = 50^\circ$.

OR

Because $\triangle ABC$ and $\triangle ADC$ are isosceles triangles and $\overline{BD}$ bisects $\angle ABC$ and $\angle ADC$, applying the Exterior Angle Theorem to $\triangle ABD$ gives $\angle BAD = 70^\circ - 20^\circ = 50^\circ$.

2017B

8. **Answer (C):** The altitude $\overline{AD}$ lies on a line of symmetry for the isosceles triangle. Under reflection about this line, B will be sent to C. Because B is obtained from D by adding 3 to the x-coordinate and subtracting 6 from the y-coordinate, C is obtained from D by subtracting 3 from the x-coordinate and adding 6 to the y-coordinate. Thus the third vertex C has coordinates $(-1-3, 3+6) = (-4, 9)$.

OR

To find the coordinates of $C(x, y)$, note that D is the midpoint of $\overline{BC}$. Therefore

$$\frac{x+2}{2} = -1 \qquad \text{and} \qquad \frac{y-3}{2} = 3.$$

Solving these equations gives $x = -4$ and $y = 9$, so $C = (-4, 9)$.

2014A

9. **Answer (C):** The area of the triangle is $\frac{1}{2} \cdot 2\sqrt{3} \cdot 6 = 6\sqrt{3}$. By the Pythagorean Theorem, the hypotenuse has length $4\sqrt{3}$. The desired altitude has length $\frac{6\sqrt{3}}{\frac{1}{2} \cdot 4\sqrt{3}} = 3$.

2016B

9. **Answer (C):** Let the vertex of the triangle that lies in the first quadrant be (x, x^2). Then the base of the triangle is $2x$ and the height is x^2, so $\frac{1}{2} \cdot 2x \cdot x^2 = 64$. Thus $x^3 = 64$, $x = 4$, and $BC = 2x = 8$.

2000

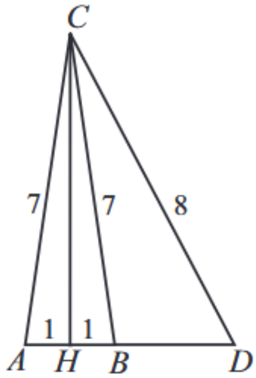
10. **Answer (D):** By the *Triangle Inequality*, each of x and y can be any number strictly between 2 and 10, so $0 \leq |x-y| < 8$. Therefore, the smallest positive number that is not a possible value of $|x-y|$ is $10-2=8$.

2005B

10. **(A)** Let $\overline{CH}$ be an altitude of $\triangle ABC$. Applying the Pythagorean Theorem to $\triangle CHB$ and to $\triangle CHD$ produces

$$8^2-(BD+1)^2=CH^2=7^2-1^2=48, \quad \text{so} \quad (BD+1)^2=16.$$

Thus $BD=3$.

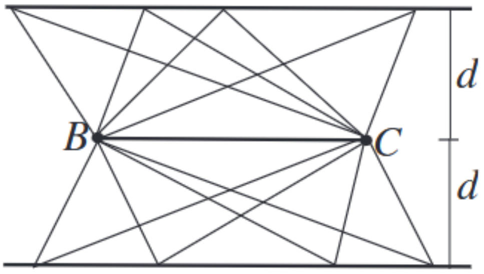


2006B

10. **(A)** Let the sides of the triangle have lengths x, $3x$, and 15. The Triangle Inequality implies that $3x < x+15$, so $x < 7.5$. Because x is an integer, the greatest possible perimeter is $7+21+15=43$.

2007B

10. **Answer (A):** If the altitude from A has length d, then $\triangle ABC$ has area $(1/2)(BC)d$. The area is 1 if and only if $d = 2/(BC)$. Thus S consists of the two lines that are parallel to line BC and are $2/(BC)$ units from it, as shown.

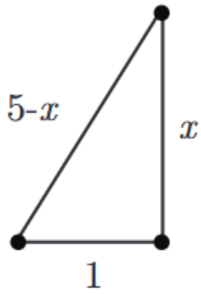


2009B

10. **Answer (E):** Let x be the height of the stump. Then $5 - x$ is the height of the snapped part, now forming the hypotenuse of a right triangle. By the Pythagorean Theorem,

$$x^2 + 1^2 = (5 - x)^2 = x^2 - 10x + 25$$

from which $x = 2.4$.



2016B

10. **Answer (D):** The weight of an object of uniform density is proportional to its volume. The volume of the triangular piece of wood of uniform thickness is proportional to the area of the triangle. The side length of the second piece is $\frac{5}{3}$ times the side length of the first piece, so the area of the second piece is $\left(\frac{5}{3}\right)^2$ times the area of the first piece. Therefore the weight is $12 \cdot \left(\frac{5}{3}\right)^2 = \frac{100}{3} \approx 33.3$ ounces.