9

COMBINATIONS

2016B

6. **Answer (B):** Because S has to be greater than 300, the digit sum has to be at least 4, and an example like $197 + 203 = 400$ shows that 4 is indeed the smallest possible value.

2013A

7. **Answer (C):** Because English is required, the student must choose 3 of the remaining 5 courses. If the student takes both math courses, there are 3 possible choices for the final course. If the student chooses only one of the 2 possible

math courses, then the student must omit one of the 3 remaining courses, for a total of $2 \cdot 3 = 6$ programs. Hence there are $3 + 6 = 9$ programs.

OR

Because English is required, there are 5 remaining courses from which a student must choose 3. Of those $\binom{5}{3}$ possibilities, one does not include a math course. Thus the number of possible programs is $\binom{5}{3} - 1 = 9$.

2018A

7. **Answer (E):** Because $4000 = 2^5 \cdot 5^3$,

$$4000 \cdot \left(\frac{2}{5}\right)^n = 2^{5+n} \cdot 5^{3-n}.$$

This product will be an integer if and only if both of the factors 2^{5+n} and 5^{3-n} are integers, which happens if and only if both exponents are nonnegative. Therefore the given expression is an integer if and only if $5+n \geq 0$ and $3-n \geq 0$. The solutions are exactly the integers satisfying $-5 \leq n \leq 3$. There are $3-(-5)+1 = 9$ such values.

2007B

8. **Answer (D):** Once a and c are chosen, the integer b is determined. For $a = 0$, we could have $c = 2$, 4, 6, or 8. For $a = 2$, we could have $c = 4$, 6, or 8. For $a = 4$, we could have $c = 6$ or 8, and for $a = 6$ the only possibility is $c = 8$. Thus there are $1+2+3+4 = 10$ possibilities when a is even. Similarly, there are 10 possibilities when a is odd, so the number of possibilities is 20.

2010B

8. **Answer (E):** The cost of an individual ticket must divide 48 and 64. The common factors of 48 and 64 are $1, 2, 4, 8$, and 16. Each of these may be the cost of one ticket, so there are 5 possible values for x.

2012B

8. **Answer (D):** Let the three whole numbers be $a < b < c$. The set of sums of pairs of these numbers is $(a+b, a+c, b+c) = (12, 17, 19)$. Thus $2(a+b+c) = (a+b)+(a+c)+(b+c) = 12+17+19 = 48$, and $a+b+c = 24$. If follows that $(a, b, c) = (24-19, 24-17, 24-12) = (5, 7, 12)$. Therefore the middle number is 7.

2017A

8. **Answer (B):** Each of the 20 people who know each other shakes hands with 10 people. Each of the 10 people who know no one shakes hands with 29 people. Because each handshake involves two people, the number of handshakes is $\frac{1}{2}(20 \cdot 10 + 10 \cdot 29) = 245$.

2012B

9. **Answer (D):** The sum could be 7 only if the even die showed 2 and the odd showed 5, the even showed 4 and the odd showed 3, or the even showed 6 and the odd showed 1. Each of these events can occur in $2 \cdot 2 = 4$ ways. Hence there are 12 ways for a 7 to occur. There are $6 \cdot 6 = 36$ possible outcomes, so the probability that a 7 occurs is $\frac{12}{36} = \frac{1}{3}$.

2002B

9. **(D)** The last "word," which occupies position 120, is *USOMA*. Immediately preceding this we have *USOAM*, *USMOA*, *USMAO*, *USAOM*,and *USAMO*. The alphabetic position of the word *USAMO* is consequently 115.

2003B

10. (C) In the old scheme 26×10^4 different plates could be constructed. In the new scheme $26^3 \times 10^3$ different plates can be constructed. There are

$$\frac{26^3 \times 10^3}{26 \times 10^4} = \frac{26^2}{10}$$

times as many possible plates with the new scheme.

2014B

10. **Answer (C):** As indicated by the leftmost column $A + B \leq 9$. Then both the second and fourth columns show that $C = 0$. Because A, B, and C are distinct digits, D must be at least 3. The following values for (A, B, C, D) show that D may be any of the 7 digits that are at least 3: $(1,2,0,3)$, $(1,3,0,4)$, $(2,3,0,5)$, $(2,4,0,6)$, $(2,5,0,7)$, $(2,6,0,8)$, $(2,7,0,9)$.

2015A 10. **Answer (C):** In the alphabet the letter b is adjacent to both a and c. So in any rearrangement, b can only be adjacent to d, and thus b must be the first or last letter in the rearrangement. Similarly, the letter c can only be adjacent to a, so c must be the first or last letter in the rearrangement. Thus the only two acceptable rearrangements are $bdac$ and $cadb$.