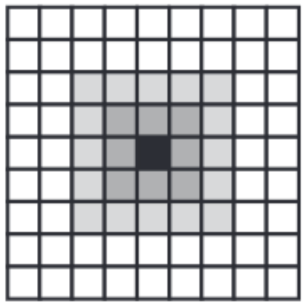
1

2D GEOMETERY

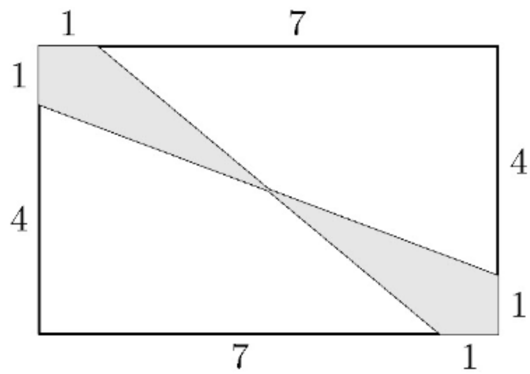
2001

11. Consider the dark square in an array of unit squares, part of which is shown. The first ring of squares around this center square contains 8 unit squares. The second ring contains 16 unit squares. If we continue this process, the number of unit squares in the 100^{th} ring is



(A) 396 **(B)** 404 **(C)** 800 **(D)** 10,000 **(E)** 10,404

2016A 11. What is the area of the shaded region of the given 8×5 rectangle?



(A) $4\frac{3}{4}$ **(B)** 5 **(C)** $5\frac{1}{4}$ **(D)** $6\frac{1}{2}$ **(E)** 8

2000

12. Figure 0,1,2, and 3 consist of 1,5,13, and 25 nonoverlapping unit squares, respectively. If the pattern were continued, how many nonoverlapping unit squares would there be in figure 100?

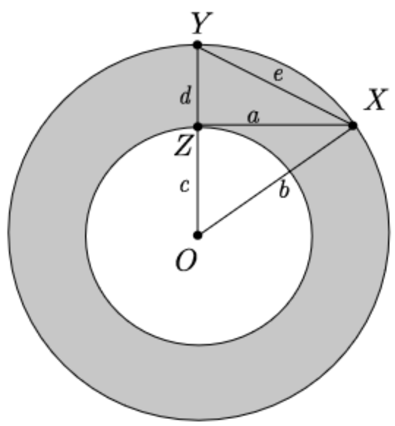
figure 0 figure 1 figure 2 figure 3

(A) 10401 **(B)** 19801 **(C)** 20201 **(D)** 39801 **(E)** 40801

2004B

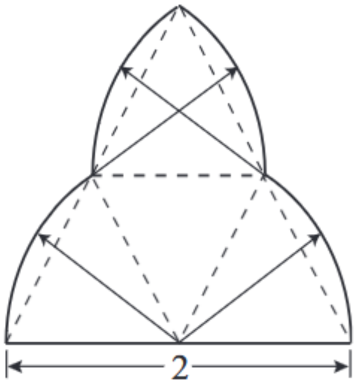
12. An *annulus* is the region between two concentric circles. The concentric circles in the figure have radii b and c, with $b > c$. Let $\overline{OX}$ be a radius of the larger circle, let $\overline{XZ}$ be tangent to the smaller circle at Z, and let $\overline{OY}$ be the radius of the larger circle that contains Z. Let $a = XZ$, $d = YZ$, and $e = XY$. What is the area of the annulus?

(A) πa^2 **(B)** πb^2 **(C)** πc^2 **(D)** πd^2 **(E)** πe^2



2005A

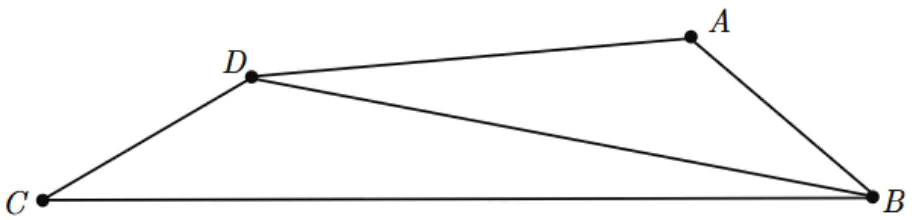
12. The figure shown is called a *trefoil* and is constructed by drawing circular sectors about sides of the congruent equilateral triangles. What is the area of a trefoil whose horizontal base has length 2?



(A) $\frac{1}{3}\pi + \frac{\sqrt{3}}{2}$ **(B)** $\frac{2}{3}\pi$ **(C)** $\frac{2}{3}\pi + \frac{\sqrt{3}}{4}$ **(D)** $\frac{2}{3}\pi + \frac{\sqrt{3}}{3}$ **(E)** $\frac{2}{3}\pi + \frac{\sqrt{3}}{2}$

2009A

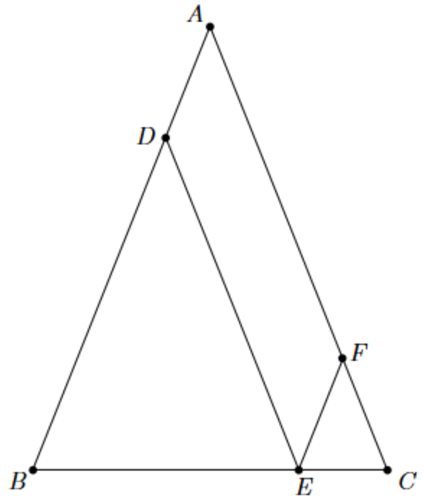
12. In quadrilateral $ABCD$, $AB = 5$, $BC = 17$, $CD = 5$, $DA = 9$, and BD is an integer. What is BD?



(A) 11 **(B)** 12 **(C)** 13 **(D)** 14 **(E)** 15

2013A

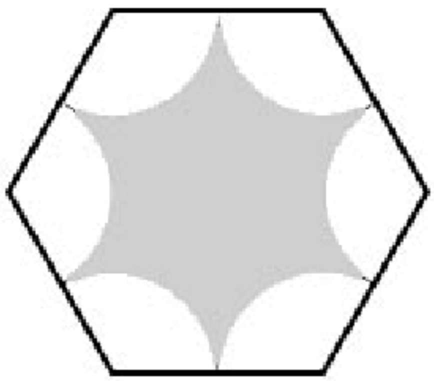
12. In $\triangle ABC$, $AB = AC = 28$ and $BC = 20$. Points D, E, and F are on sides $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$, respectively, such that $\overline{DE}$ and $\overline{EF}$ are parallel to $\overline{AC}$ and $\overline{AB}$, respectively. What is the perimeter of parallelogram $ADEF$?



(A) 48 **(B)** 52 **(C)** 56 **(D)** 60 **(E)** 72

2014A

12. A regular hexagon has side length 6. Congruent arcs with radius 3 are drawn with the center at each of the vertices, creating circular sectors as shown. The region inside the hexagon but outside the sectors is shaded as shown. What is the area of the shaded region?

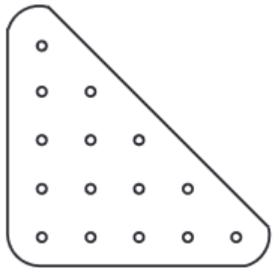


(A) $27\sqrt{3} - 9\pi$ **(B)** $27\sqrt{3} - 6\pi$ **(C)** $54\sqrt{3} - 18\pi$

(D) $54\sqrt{3} - 12\pi$ **(E)** $108\sqrt{3} - 9\pi$

2000

13. There are 5 yellow pegs, 4 red pegs, 3 green pegs, 2 blue pegs, and 1 orange peg to be placed on a triangular peg board. In how many ways can the pegs be placed so that no (horizontal) row or (vertical) column contains two pegs of the same color?

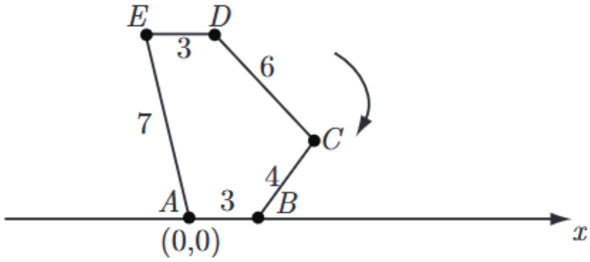


(A) 0 **(B)** 1 **(C)** $5! \cdot 4! \cdot 3! \cdot 2! \cdot 1!$

(D) $15!/(5! \cdot 4! \cdot 3! \cdot 2! \cdot 1!)$ **(E)** $15!$

2009B

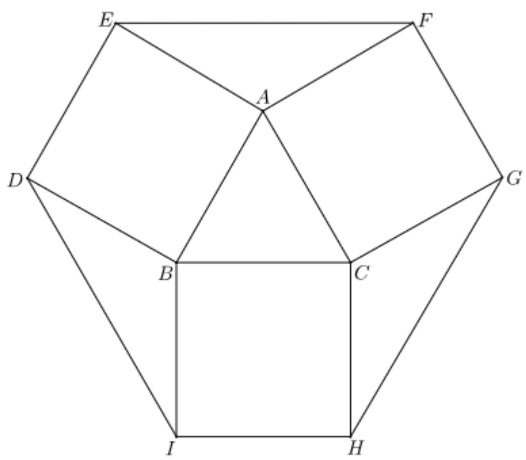
13. As shown below, convex pentagon $ABCDE$ has sides $AB = 3$, $BC = 4$, $CD = 6$, $DE = 3$, and $EA = 7$. The pentagon is originally positioned in the plane with vertex A at the origin and vertex B on the positive x-axis. The pentagon is then rolled clockwise to the right along the x-axis. Which side will touch the point $x = 2009$ on the x-axis?



(A) $\overline{AB}$ **(B)** $\overline{BC}$ **(C)** $\overline{CD}$ **(D)** $\overline{DE}$ **(E)** $\overline{EA}$

2014A

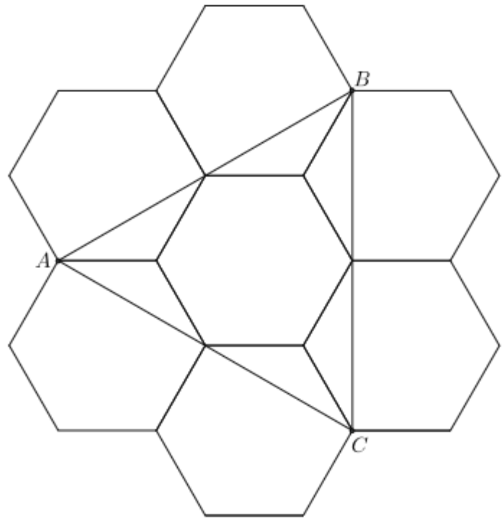
13. Equilateral $\triangle ABC$ has side length 1, and squares $ABDE$, $BCHI$, and $CAFG$ lie outside the triangle. What is the area of hexagon $DEFGHI$?



(A) $\dfrac{12+3\sqrt{3}}{4}$ **(B)** $\dfrac{9}{2}$ **(C)** $3+\sqrt{3}$ **(D)** $\dfrac{6+3\sqrt{3}}{2}$ **(E)** 6

2014B

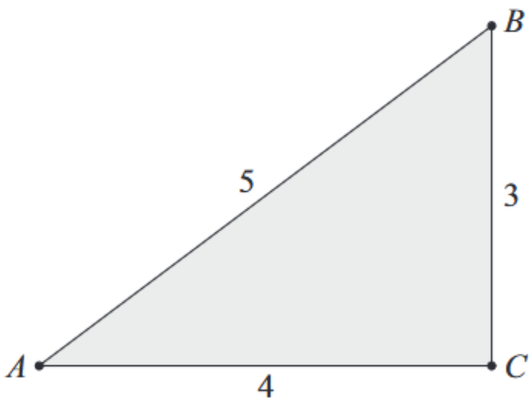
13. Six regular hexagons surround a regular hexagon of side length 1 as shown. What is the area of $\triangle ABC$?



(A) $2\sqrt{3}$ **(B)** $3\sqrt{3}$ **(C)** $1+3\sqrt{2}$ **(D)** $2+2\sqrt{3}$ **(E)** $3+2\sqrt{3}$

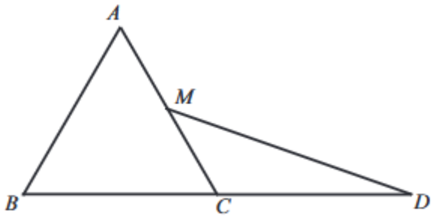
2018A

13. A paper triangle with sides of lengths 3, 4, and 5 inches, as shown, is folded so that point A falls on point B. What is the length in inches of the crease?



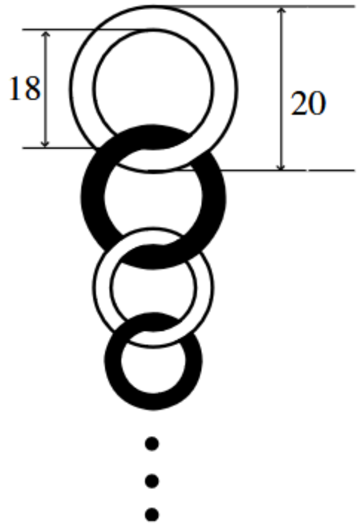
(A) $1+\frac{1}{2}\sqrt{2}$ **(B)** $\sqrt{3}$ **(C)** $\frac{7}{4}$ **(D)** $\frac{15}{8}$ **(E)** 2

2005B 14. Equilateral $\triangle ABC$ has side length 2, M is the midpoint of $\overline{AC}$, and C is the midpoint of $\overline{BD}$. What is the area of $\triangle CDM$?



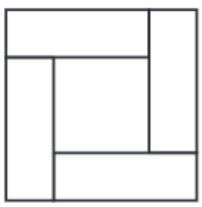
(A) $\frac{\sqrt{2}}{2}$ **(B)** $\frac{3}{4}$ **(C)** $\frac{\sqrt{3}}{2}$ **(D)** 1 **(E)** $\sqrt{2}$

2006A 14. A number of linked rings, each 1 cm thick, are hanging on a peg. The top ring has an outside diameter of 20 cm. The outside diameter of each of the other rings is 1 cm less than that of the ring above it. The bottom ring has an outside diameter of 3 cm. What is the distance, in cm, from the top of the top ring to the bottom of the bottom ring?



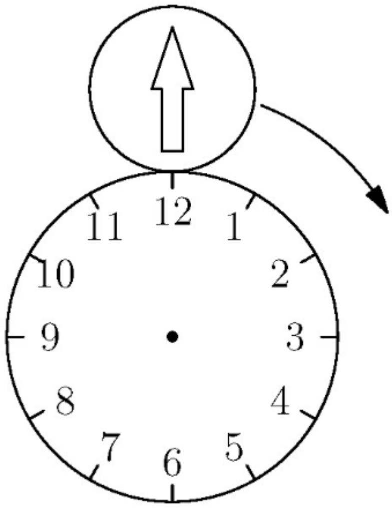
(A) 171 **(B)** 173 **(C)** 182 **(D)** 188 **(E)** 210

2009A 14. Four congruent rectangles are placed as shown. The area of the outer square is 4 times that of the inner square. What is the ratio of the length of the longer side of each rectangle to the length of its shorter side?



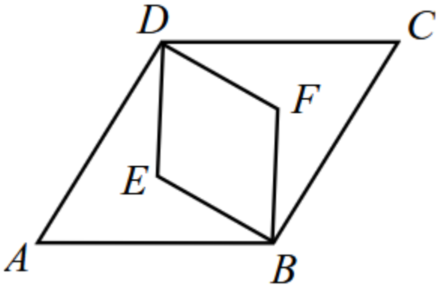
(A) 3 **(B)** $\sqrt{10}$ **(C)** $2+\sqrt{2}$ **(D)** $2\sqrt{3}$ **(E)** 4

2015A 14. The diagram below shows the circular face of a clock with radius 20 cm and a circular disk with radius 10 cm externally tangent to the clock face at 12 o'clock. The disk has an arrow painted on it, initially pointing in the upward vertical direction. Let the disk roll clockwise around the clock face. At what point on the clock face will the disk be tangent when the arrow is next pointing in the upward vertical direction?



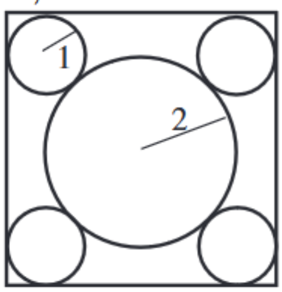
(A) 2 o'clock **(B)** 3 o'clock **(C)** 4 o'clock **(D)** 6 o'clock **(E)** 8 o'clock

2006B 15. Rhombus $ABCD$ is similar to rhombus $BFDE$. The area of rhombus $ABCD$ is 24, and $\angle BAD = 60°$. What is the area of rhombus $BFDE$?



(A) 6 **(B)** $4\sqrt{3}$ **(C)** 8 **(D)** 9 **(E)** $6\sqrt{3}$

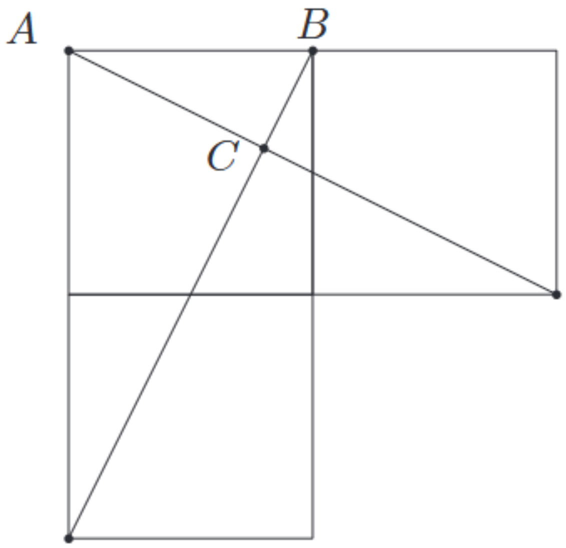
2007A 15. Four circles of radius 1 are each tangent to two sides of a square and externally tangent to a circle of radius 2, as shown. What is the area of the square?



(A) 32 **(B)** $22 + 12\sqrt{2}$ **(C)** $16 + 16\sqrt{3}$ **(D)** 48 **(E)** $36 + 16\sqrt{2}$

2012A

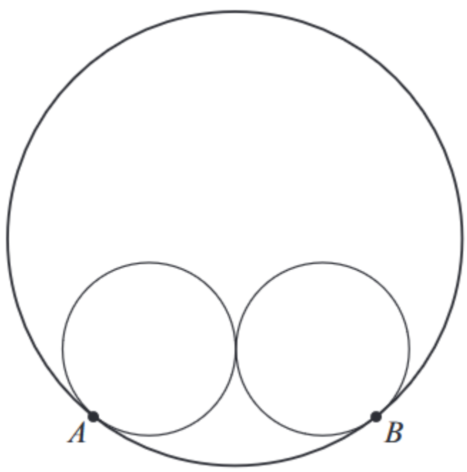
15. Three unit squares and two line segments connecting two pairs of vertices are shown. What is the area of $\triangle ABC$?



(A) $\frac{1}{6}$ **(B)** $\frac{1}{5}$ **(C)** $\frac{2}{9}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{\sqrt{2}}{4}$

2018A

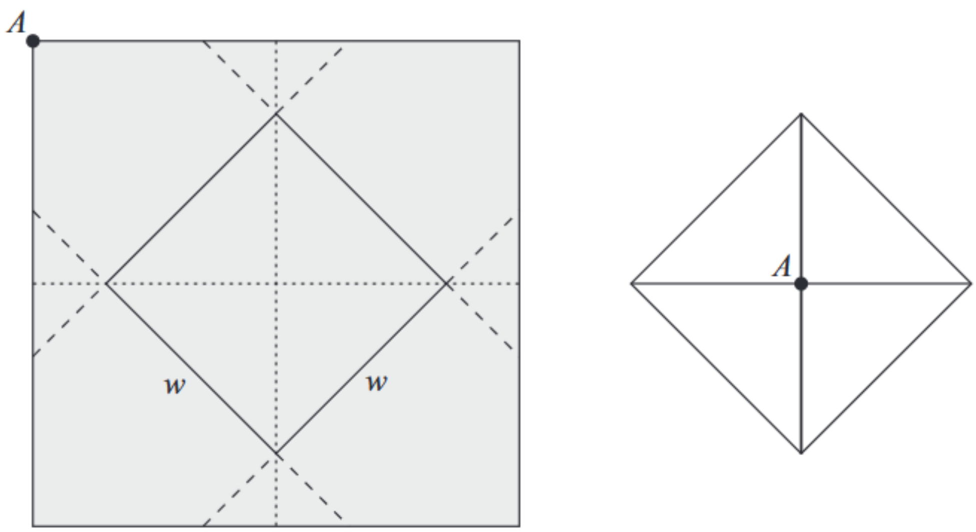
15. Two circles of radius 5 are externally tangent to each other and are internally tangent to a circle of radius 13 at points A and B, as shown in the diagram. The distance AB can be written in the form $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m+n$?



(A) 21 **(B)** 29 **(C)** 58 **(D)** 69 **(E)** 93

2018B

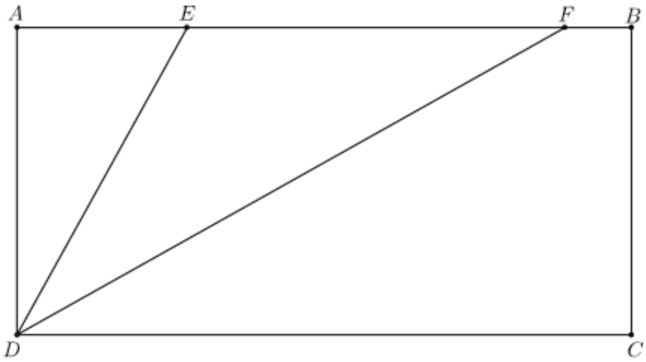
15. A closed box with a square base is to be wrapped with a square sheet of wrapping paper. The box is centered on the wrapping paper with the vertices of the base lying on the midlines of the square sheet of paper, as shown in the figure on the left. The four corners of the wrapping paper are to be folded up over the sides and brought together to meet at the center of the top of the box, point A in the figure on the right. The box has base length w and height h. What is the area of the sheet of wrapping paper?



(A) $2(w+h)^2$ **(B)** $\frac{(w+h)^2}{2}$ **(C)** $2w^2+4wh$ **(D)** $2w^2$

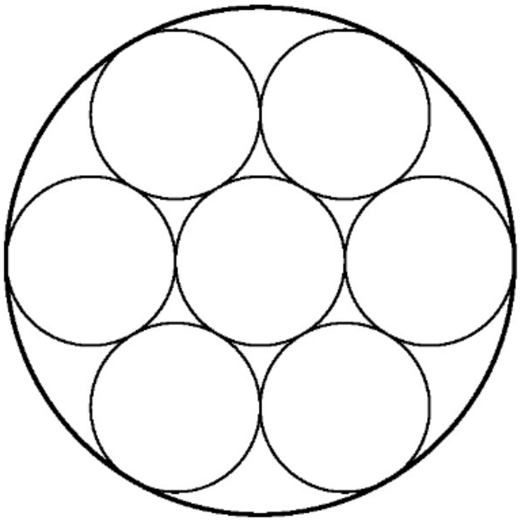
(E) w^2h

2014B 15. In rectangle $ABCD$, $DC = 2CB$ and points E and F lie on $\overline{AB}$ so that $\overline{ED}$ and $\overline{FD}$ trisect $\angle ADC$ as shown. What is the ratio of the area of $\triangle DEF$ to the area of rectangle $ABCD$?



(A) $\frac{\sqrt{3}}{6}$ **(B)** $\frac{\sqrt{6}}{8}$ **(C)** $\frac{3\sqrt{3}}{16}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{\sqrt{2}}{4}$

2016A 15. Seven cookies of radius 1 inch are cut from a circle of cookie dough, as shown. Neighboring cookies are tangent, and all except the center cookie are tangent to the edge of the dough. The leftover scrap is reshaped to form another cookie of the same thickness. What is the radius in inches of the scrap cookie?



(A) $\sqrt{2}$ **(B)** 1.5 **(C)** $\sqrt{\pi}$ **(D)** $\sqrt{2\pi}$ **(E)** π

2