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## PROBABILITY

- 2015B 11. Among the positive integers less than 100, each of whose digits is a prime number, one is selected at random. What is the probability that the selected number is prime?
- (A)  $\frac{8}{99}$       (B)  $\frac{2}{5}$       (C)  $\frac{9}{20}$       (D)  $\frac{1}{2}$       (E)  $\frac{9}{16}$

- 2004B 11. Two eight-sided dice each have faces numbered 1 through 8. When the dice are rolled, each face has an equal probability of appearing on the top. What is the probability that the product of the two top numbers is greater than their sum?
- (A)  $\frac{1}{2}$       (B)  $\frac{47}{64}$       (C)  $\frac{3}{4}$       (D)  $\frac{55}{64}$       (E)  $\frac{7}{8}$

- 2018A 11. When 7 fair standard 6-sided dice are thrown, the probability that the sum of the numbers on the top faces is 10 can be written as

$$\frac{n}{6^7},$$

where  $n$  is a positive integer. What is  $n$ ?

- (A) 42      (B) 49      (C) 56      (D) 63      (E) 84

- 2003A 12. A point  $(x, y)$  is randomly picked from inside the rectangle with vertices  $(0, 0)$ ,  $(4, 0)$ ,  $(4, 1)$ , and  $(0, 1)$ . What is the probability that  $x < y$ ?
- (A)  $\frac{1}{8}$       (B)  $\frac{1}{4}$       (C)  $\frac{3}{8}$       (D)  $\frac{1}{2}$       (E)  $\frac{3}{4}$

- 2005B 12. Twelve fair dice are rolled. What is the probability that the **product** of the numbers on the top faces is prime?
- (A)  $\left(\frac{1}{12}\right)^{12}$     (B)  $\left(\frac{1}{6}\right)^{12}$     (C)  $2\left(\frac{1}{6}\right)^{11}$     (D)  $\frac{5}{2}\left(\frac{1}{6}\right)^{11}$     (E)  $\left(\frac{1}{6}\right)^{10}$
- 2013B 12. Let  $S$  be the set of sides and diagonals of a regular pentagon. A pair of elements of  $S$  are selected at random without replacement. What is the probability that the two chosen segments have the same length?
- (A)  $\frac{2}{5}$     (B)  $\frac{4}{9}$     (C)  $\frac{1}{2}$     (D)  $\frac{5}{9}$     (E)  $\frac{4}{5}$
- 2016A 12. Three distinct integers are selected at random between 1 and 2016, inclusive. Which of the following is a correct statement about the probability  $p$  that the product of the three integers is odd?
- (A)  $p < \frac{1}{8}$     (B)  $p = \frac{1}{8}$     (C)  $\frac{1}{8} < p < \frac{1}{3}$     (D)  $p = \frac{1}{3}$     (E)  $p > \frac{1}{3}$

- 2016B 12. Two different numbers are selected at random from  $\{1, 2, 3, 4, 5\}$  and multiplied together. What is the probability that the product is even?
- (A) 0.2    (B) 0.4    (C) 0.5    (D) 0.7    (E) 0.8
- 2006A 13. A player pays \$5 to play a game. A die is rolled. If the number on the die is odd, the game is lost. If the number on the die is even, the die is rolled again. In this case the player wins if the second number matches the first and loses otherwise. How much should the player win if the game is fair? (In a fair game the probability of winning times the amount won is what the player should pay.)
- (A) \$12    (B) \$30    (C) \$50    (D) \$60    (E) \$100
- 2011B 13. Two real numbers are selected independently at random from the interval  $[-20, 10]$ . What is the probability that the product of those numbers is greater than zero?
- (A)  $\frac{1}{9}$     (B)  $\frac{1}{3}$     (C)  $\frac{4}{9}$     (D)  $\frac{5}{9}$     (E)  $\frac{2}{3}$

- 2017B 14. An integer  $N$  is selected at random in the range  $1 \leq N \leq 2020$ . What is the probability that the remainder when  $N^{16}$  is divided by 5 is 1?
- (A)  $\frac{1}{5}$       (B)  $\frac{2}{5}$       (C)  $\frac{3}{5}$       (D)  $\frac{4}{5}$       (E) 1
- 2003A 15. What is the probability that an integer in the set  $\{1, 2, 3, \dots, 100\}$  is divisible by 2 and not divisible by 3?
- (A)  $\frac{1}{6}$       (B)  $\frac{33}{100}$       (C)  $\frac{17}{50}$       (D)  $\frac{1}{2}$       (E)  $\frac{18}{25}$
- 2005B 15. An envelope contains eight bills: 2 ones, 2 fives, 2 tens, and 2 twenties. Two bills are drawn at random without replacement. What is the probability that their sum is \$20 or more?
- (A)  $\frac{1}{4}$       (B)  $\frac{2}{5}$       (C)  $\frac{3}{7}$       (D)  $\frac{1}{2}$       (E)  $\frac{2}{3}$

- 2017A 15. Chloé chooses a real number uniformly at random from the interval  $[0, 2017]$ . Independently, Laurent chooses a real number uniformly at random from the interval  $[0, 4034]$ . What is the probability that Laurent's number is greater than Chloé's number?

(A)  $\frac{1}{2}$       (B)  $\frac{2}{3}$       (C)  $\frac{3}{4}$       (D)  $\frac{5}{6}$       (E)  $\frac{7}{8}$