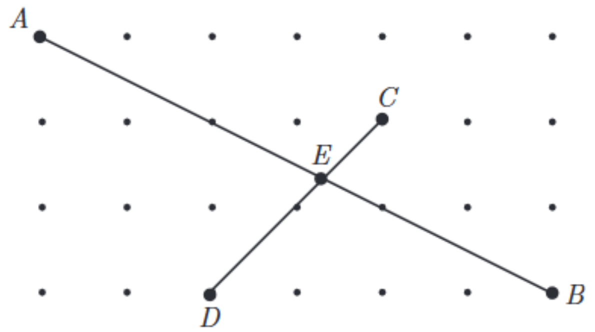
1

2D GEOMTERY

2000

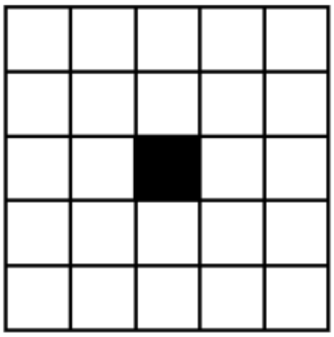
16. The diagram shows 28 lattice points, each one unit from its nearest neighbors. Segment AB meets segment CD at E. Find the length of segment AE.



(A) $4\sqrt{5}/3$ (B) $5\sqrt{5}/3$ (C) $12\sqrt{5}/7$ (D) $2\sqrt{5}$ (E) $5\sqrt{65}/9$

2004A

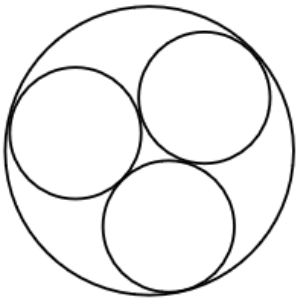
16. The 5×5 grid shown contains a collection of squares with sizes from 1×1 to 5×5. How many of these squares contain the black center square?



(A) 12 **(B)** 15 **(C)** 17 **(D)** 19 **(E)** 20

2004B

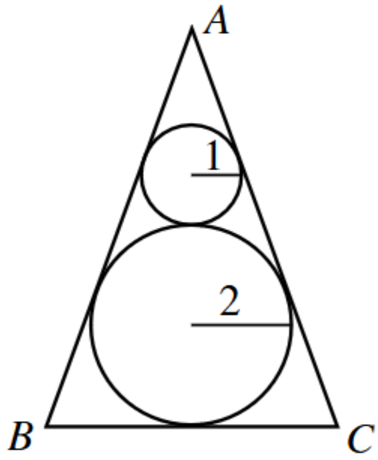
16. Three circles of radius 1 are externally tangent to each other and internally tangent to a larger circle. What is the radius of the large circle?



(A) $\frac{2+\sqrt{6}}{3}$ **(B)** 2 **(C)** $\frac{2+3\sqrt{2}}{3}$ **(D)** $\frac{3+2\sqrt{3}}{3}$ **(E)** $\frac{3+\sqrt{3}}{2}$

2006A

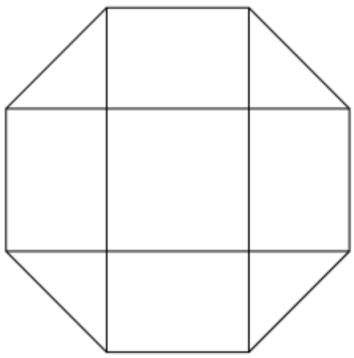
16. A circle of radius 1 is tangent to a circle of radius 2. The sides of $\triangle ABC$ are tangent to the circles as shown, and the sides $\overline{AB}$ and $\overline{AC}$ are congruent. What is the area of $\triangle ABC$?



(A) $\frac{35}{2}$ **(B)** $15\sqrt{2}$ **(C)** $\frac{64}{3}$ **(D)** $16\sqrt{2}$ **(E)** 24

2011B

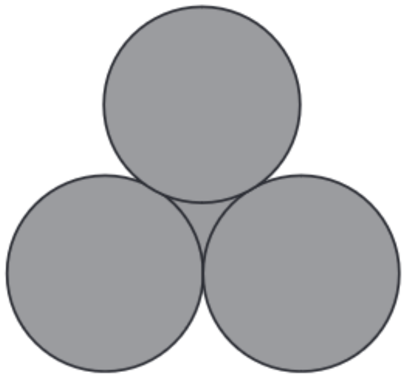
16. A dart board is a regular octagon divided into regions as shown. Suppose that a dart thrown at the board is equally likely to land anywhere on the board. What is the probability that the dart lands within the center square?



(A) $\frac{\sqrt{2}-1}{2}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{2-\sqrt{2}}{2}$ **(D)** $\frac{\sqrt{2}}{4}$ **(E)** $2-\sqrt{2}$

2012B

16. Three circles with radius 2 are mutually tangent. What is the total area of the circles and the region bounded by them, as shown in the figure?

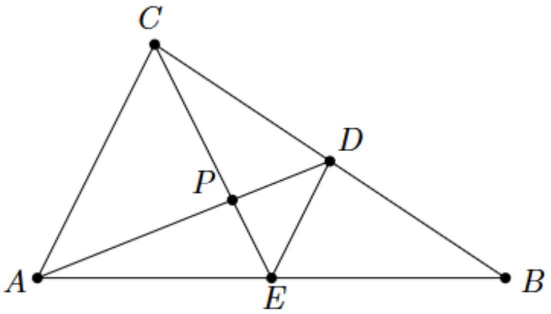


(A) $10\pi + 4\sqrt{3}$ **(B)** $13\pi - \sqrt{3}$ **(C)** $12\pi + \sqrt{3}$ **(D)** $10\pi + 9$

(E) 13π

2013B

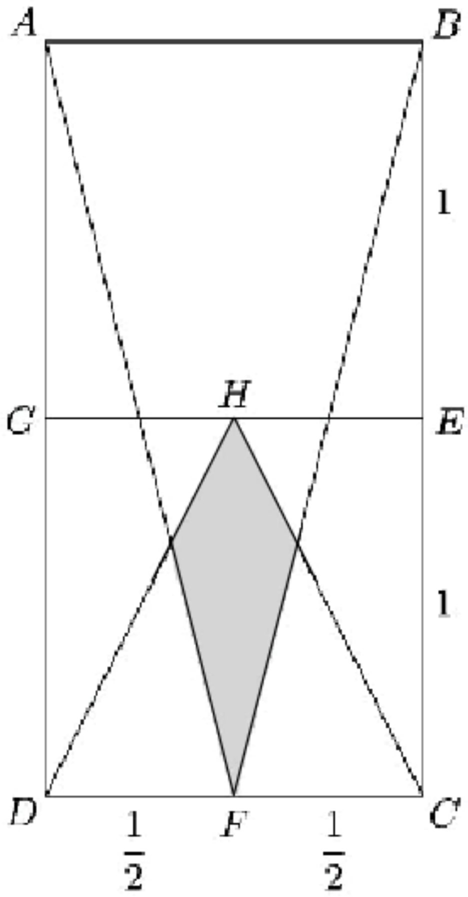
16. In $\triangle ABC$, medians $\overline{AD}$ and $\overline{CE}$ intersect at P, $PE = 1.5$, $PD = 2$, and $DE = 2.5$. What is the area of $AEDC$?



(A) 13 **(B)** 13.5 **(C)** 14 **(D)** 14.5 **(E)** 15

2014A

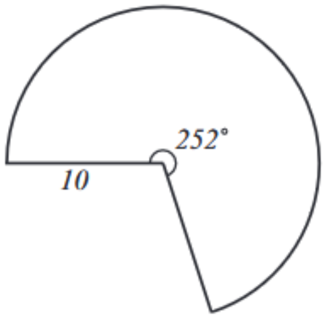
16. In rectangle $ABCD$, $AB = 1, BC = 2$, and points E, F, and G are midpoints of $\overline{BC}$, $\overline{CD}$, and $\overline{AD}$, respectively. Point H is the midpoint of $\overline{GE}$. What is the area of the shaded region?

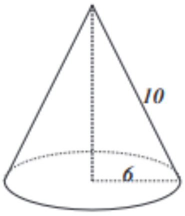


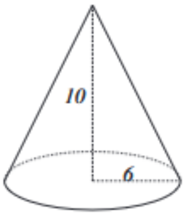
(A) $\frac{1}{12}$ **(B)** $\frac{\sqrt{3}}{18}$ **(C)** $\frac{\sqrt{2}}{12}$ **(D)** $\frac{\sqrt{3}}{12}$ **(E)** $\frac{1}{6}$

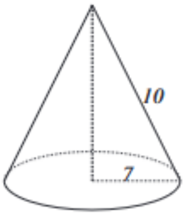
2001

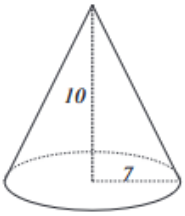
17. Which of the cones below can be formed from a 252° sector of a circle of radius 10 by aligning the two straight sides?

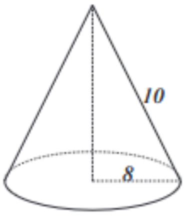


(A) 

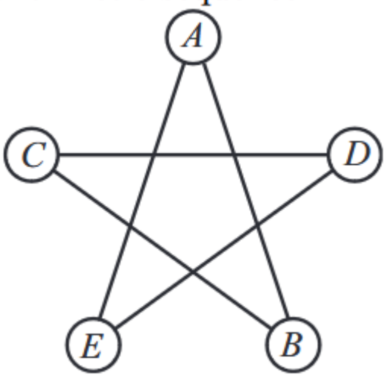
(B) 

(C) 

(D) 

(E) 

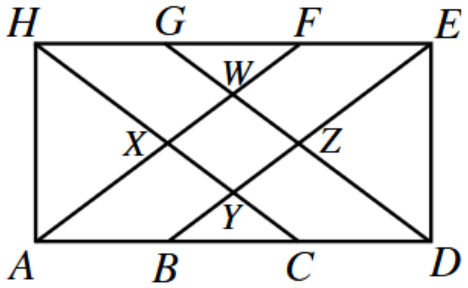
2005A 17. In the five-sided star shown, the letters A, B, C, D, and E are replaced by the numbers 3, 5, 6, 7, and 9, although not necessarily in this order. The sums of the numbers at the ends of the line segments $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DE}$, and $\overline{EA}$ form an arithmetic sequence, although not necessarily in this order. What is the middle term of the arithmetic sequence?



(A) 9 **(B)** 10 **(C)** 11 **(D)** 12 **(E)** 13

2006A

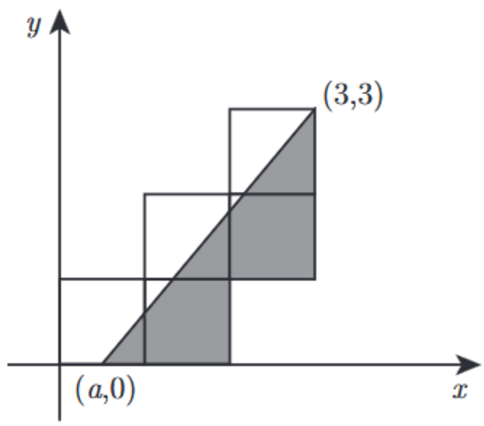
17. In rectangle $ADEH$, points B and C trisect $\overline{AD}$, and points G and F trisect $\overline{HE}$. In addition, $AH = AC = 2$. What is the area of quadrilateral $WXYZ$ shown in the figure?



(A) $\frac{1}{2}$ (B) $\frac{\sqrt{2}}{2}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{2\sqrt{2}}{3}$ (E) $\frac{2\sqrt{3}}{3}$

2009B

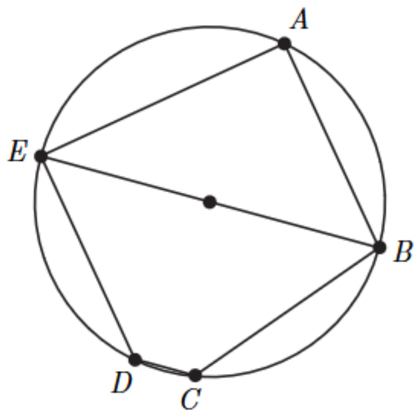
17. Five unit squares are arranged in the coordinate plane as shown, with the lower left corner at the origin. The slanted line, extending from $(a, 0)$ to $(3, 3)$, divides the entire region into two regions of equal area. What is a?



(A) $\frac{1}{2}$ (B) $\frac{3}{5}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$ (E) $\frac{4}{5}$

2011B

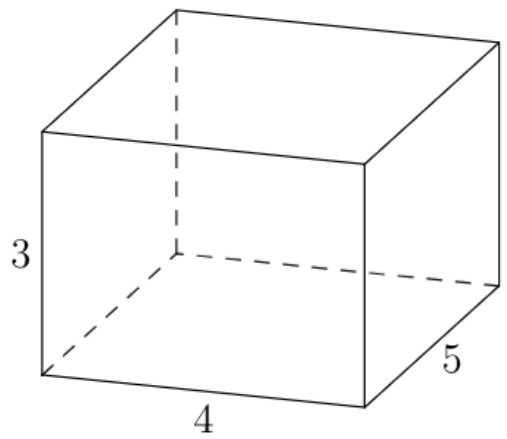
17. In the given circle, the diameter $\overline{EB}$ is parallel to $\overline{DC}$, and $\overline{AB}$ is parallel to $\overline{ED}$. The angles AEB and ABE are in the ratio $4 : 5$. What is the degree measure of angle BCD?



(A) 120 **(B)** 125 **(C)** 130 **(D)** 135 **(E)** 140

2015B

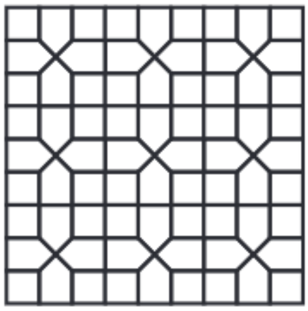
17. The centers of the faces of the right rectangular prism shown below are joined to create an octahedron. What is the volume of the octahedron?



(A) $\frac{75}{12}$ **(B)** 10 **(C)** 12 **(D)** $10\sqrt{2}$ **(E)** 15

2001

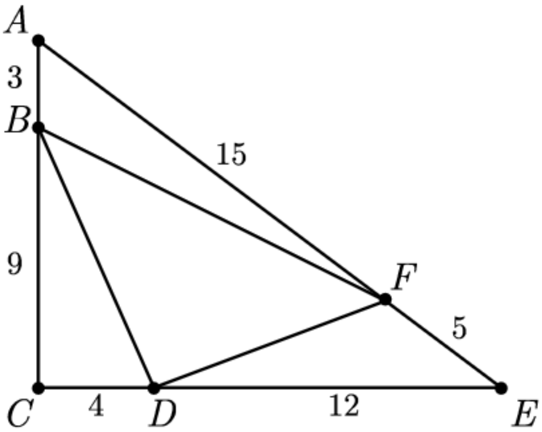
18. The plane is tiled by congruent squares and congruent pentagons as indicated. The percent of the plane that is enclosed by the pentagons is closest to



(A) 50 **(B)** 52 **(C)** 54 **(D)** 56 **(E)** 58

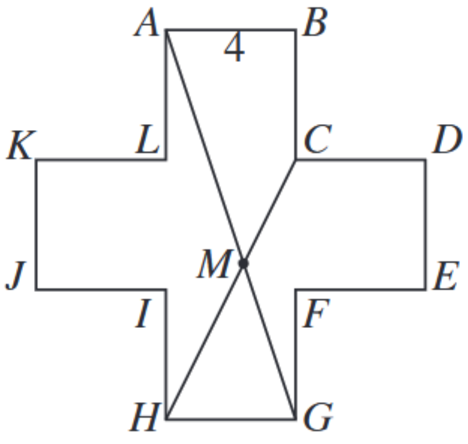
2004B

18. In right triangle $\triangle ACE$, we have $AC = 12$, $CE = 16$, and $EA = 20$. Points B, D, and F are located on $\overline{AC}$, $\overline{CE}$, and $\overline{EA}$, respectively, so that $AB = 3$, $CD = 4$, and $EF = 5$. What is the ratio of the area of $\triangle BDF$ to that of $\triangle ACE$?



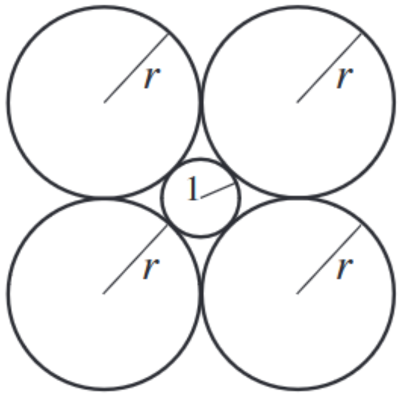
(A) $\frac{1}{4}$ **(B)** $\frac{9}{25}$ **(C)** $\frac{3}{8}$ **(D)** $\frac{11}{25}$ **(E)** $\frac{7}{16}$

2007A 18. Consider the 12-sided polygon $ABCDEFGHIJKL$, as shown. Each of its sides has length 4, and each two consecutive sides form a right angle. Suppose that $\overline{AG}$ and $\overline{CH}$ meet at M. What is the area of quadrilateral $ABCM$?



(A) $\frac{44}{3}$ **(B)** 16 **(C)** $\frac{88}{5}$ **(D)** 20 **(E)** $\frac{62}{3}$

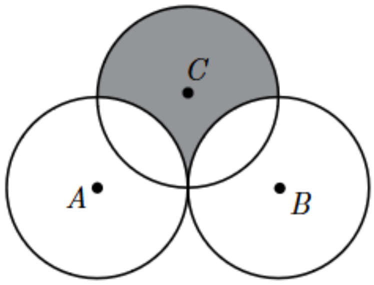
2007B 18. A circle of radius 1 is surrounded by 4 circles of radius r as shown. What is r?



(A) $\sqrt{2}$ **(B)** $1+\sqrt{2}$ **(C)** $\sqrt{6}$ **(D)** 3 **(E)** $2+\sqrt{2}$

2011A

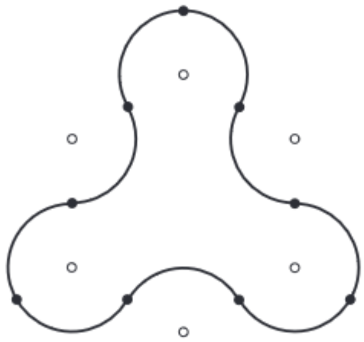
18. Circles A, B, and C each have radius 1. Circles A and B share one point of tangency. Circle C has a point of tangency with the midpoint of $\overline{AB}$. What is the area inside circle C but outside circle A and circle B?



(A) $3 - \frac{\pi}{2}$ **(B)** $\frac{\pi}{2}$ **(C)** 2 **(D)** $\frac{3\pi}{4}$ **(E)** $1 + \frac{\pi}{2}$

2012A

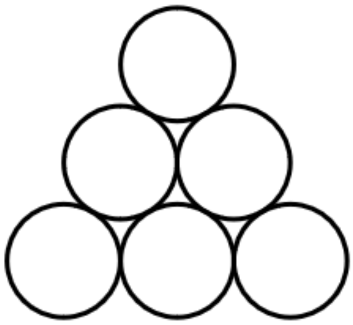
18. The closed curve in the figure is made up of 9 congruent circular arcs each of length $\frac{2\pi}{3}$, where each of the centers of the corresponding circles is among the vertices of a regular hexagon of side 2. What is the area enclosed by the curve?



(A) $2\pi + 6$ **(B)** $2\pi + 4\sqrt{3}$ **(C)** $3\pi + 4$ **(D)** $2\pi + 3\sqrt{3} + 2$ **(E)** $\pi + 6\sqrt{3}$

2017B

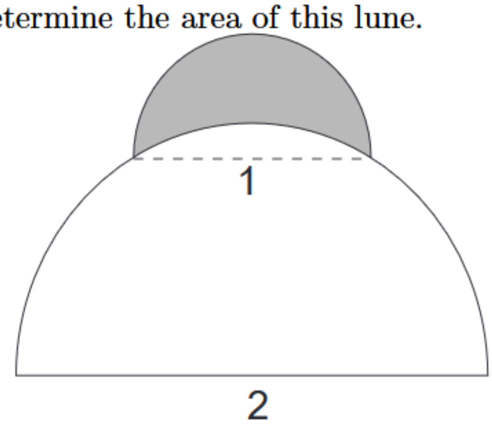
18. In the figure below, 3 of the 6 disks are to be painted blue, 2 are to be painted red, and 1 is to be painted green. Two paintings that can be obtained from one another by a rotation or a reflection of the entire figure are considered the same. How many different paintings are possible?



(A) 6 **(B)** 8 **(C)** 9 **(D)** 12 **(E)** 15

2003A

19. A semicircle of diameter 1 sits at the top of a semicircle of diameter 2, as shown. The shaded area inside the smaller semicircle and outside the larger semicircle is called a *lune*. Determine the area of this lune.

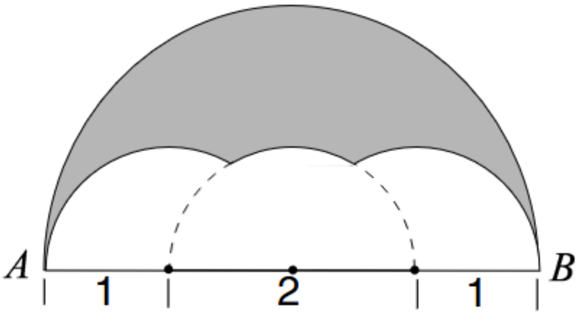


(A) $\frac{1}{6}\pi - \frac{\sqrt{3}}{4}$ **(B)** $\frac{\sqrt{3}}{4} - \frac{1}{12}\pi$ **(C)** $\frac{\sqrt{3}}{4} - \frac{1}{24}\pi$ **(D)** $\frac{\sqrt{3}}{4} + \frac{1}{24}\pi$

(E) $\frac{\sqrt{3}}{4} + \frac{1}{12}\pi$

2003B

19. Three semicircles of radius 1 are constructed on diameter $\overline{AB}$ of a semicircle of radius 2. The centers of the small semicircles divide $\overline{AB}$ into four line segments of equal length, as shown. What is the area of the shaded region that lies within the large semicircle but outside the smaller semicircles?

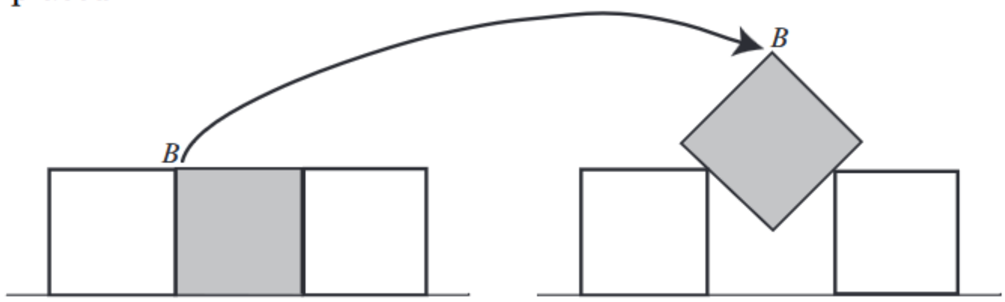


(A) $\pi - \sqrt{3}$ **(B)** $\pi - \sqrt{2}$ **(C)** $\frac{\pi + \sqrt{2}}{2}$ **(D)** $\frac{\pi + \sqrt{3}}{2}$

(E) $\frac{7}{6}\pi - \frac{\sqrt{3}}{2}$

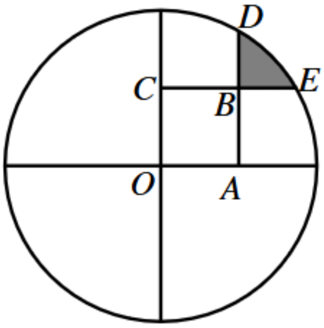
2005A

19. Three one-inch squares are placed with their bases on a line. The center square is lifted out and rotated 45°, as shown. Then it is centered and lowered into its original location until it touches both of the adjoining squares. How many inches is the point B from the line on which the bases of the original squares were placed?



(A) 1 **(B)** $\sqrt{2}$ **(C)** $\frac{3}{2}$ **(D)** $\sqrt{2} + \frac{1}{2}$ **(E)** 2

2006B 19. A circle of radius 2 is centered at O. Square $OABC$ has side length 1. Sides $\overline{AB}$ and $\overline{CB}$ are extended past B to meet the circle at D and E, respectively. What is the area of the shaded region in the figure, which is bounded by $\overline{BD}$, $\overline{BE}$, and the minor arc connecting D and E?



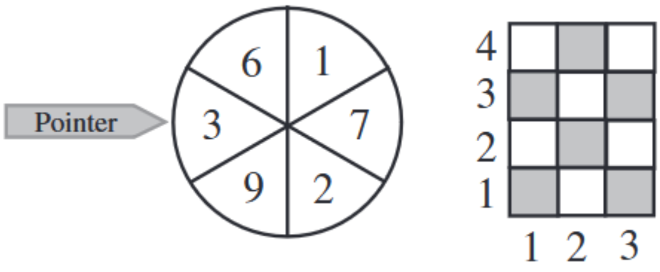
(A) $\frac{\pi}{3}+1-\sqrt{3}$ (B) $\frac{\pi}{2}(2-\sqrt{3})$ (C) $\pi(2-\sqrt{3})$ (D) $\frac{\pi}{6}+\frac{\sqrt{3}-1}{2}$ (E) $\frac{\pi}{3}-1+\sqrt{3}$

2007A 19. A paint brush is swept along both diagonals of a square to produce the symmetric painted area, as shown. Half the area of the square is painted. What is the ratio of the side length of the square to the brush width?

(A) $2\sqrt{2}+1$ (B) $3\sqrt{2}$ (C) $2\sqrt{2}+2$ (D) $3\sqrt{2}+1$ (E) $3\sqrt{2}+2$

2007B

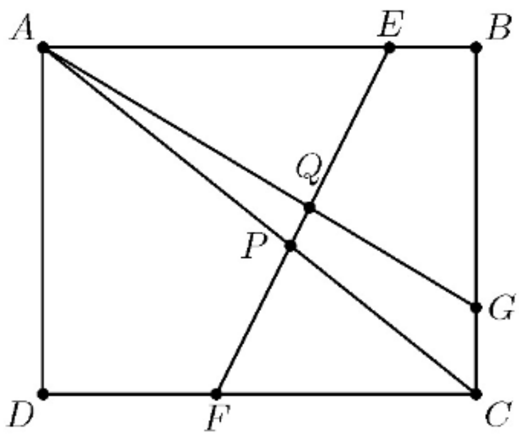
19. The wheel shown is spun twice, and the randomly determined numbers opposite the pointer are recorded. The first number is divided by 4, and the second number is divided by 5. The first remainder designates a column, and the second remainder designates a row on the checkerboard shown. What is the probability that the pair of numbers designates a shaded square?



(A) $\frac{1}{3}$ **(B)** $\frac{4}{9}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{5}{9}$ **(E)** $\frac{2}{3}$

2016B

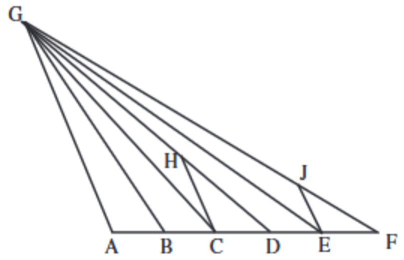
19. Rectangle $ABCD$ has $AB = 5$ and $BC = 4$. Point E lies on $\overline{AB}$ so that $EB = 1$, point G lies on $\overline{BC}$ so that $CG = 1$, and point F lies on $\overline{CD}$ so that $DF = 2$. Segments $\overline{AG}$ and $\overline{AC}$ intersect $\overline{EF}$ at Q and P, respectively. What is the value of $\frac{PQ}{EF}$?



(A) $\frac{\sqrt{3}}{16}$ **(B)** $\frac{\sqrt{2}}{13}$ **(C)** $\frac{9}{82}$ **(D)** $\frac{10}{91}$ **(E)** $\frac{1}{9}$

2002A

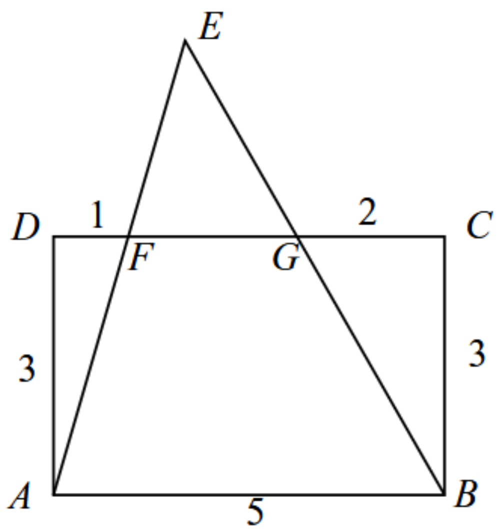
20. Points A, B, C, D, E, and F lie, in that order, on $\overline{AF}$, dividing it into five segments, each of length 1. Point G is not on line AF. Point H lies on $\overline{GD}$, and point J lies on $\overline{GF}$. The line segments $\overline{HC}$, $\overline{JE}$, and $\overline{AG}$ are parallel. Find HC/JE.



(A) 5/4 **(B)** 4/3 **(C)** 3/2 **(D)** 5/3 **(E)** 2

2003B

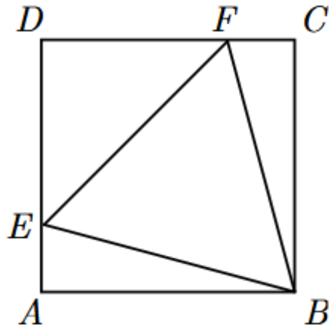
20. In rectangle $ABCD$, $AB = 5$ and $BC = 3$. Points F and G are on $\overline{CD}$ so that $DF = 1$ and $GC = 2$. Lines AF and BG intersect at E. Find the area of $\triangle AEB$.



(A) 10 **(B)** $\frac{21}{2}$ **(C)** 12 **(D)** $\frac{25}{2}$ **(E)** 15

2004A

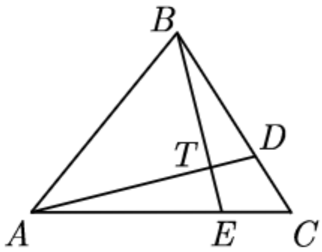
20. Points E and F are located on square $ABCD$ so that $\triangle BEF$ is equilateral. What is the ratio of the area of $\triangle DEF$ to that of $\triangle ABE$?



(A) $\frac{4}{3}$ **(B)** $\frac{3}{2}$ **(C)** $\sqrt{3}$ **(D)** 2 **(E)** $1+\sqrt{3}$

2004B

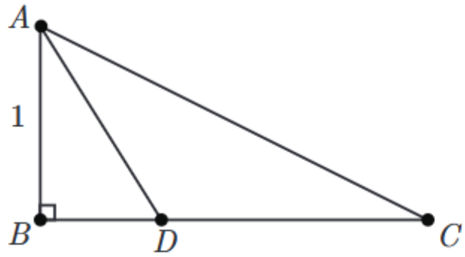
20. In $\triangle ABC$ points D and E lie on $\overline{BC}$ and $\overline{AC}$, respectively. If $\overline{AD}$ and $\overline{BE}$ intersect at T so that $AT/DT = 3$ and $BT/ET = 4$, what is CD/BD?



(A) $\frac{1}{8}$ **(B)** $\frac{2}{9}$ **(C)** $\frac{3}{10}$ **(D)** $\frac{4}{11}$ **(E)** $\frac{5}{12}$

2009B

20. Triangle ABC has a right angle at B, $AB = 1$, and $BC = 2$. The bisector of $\angle BAC$ meets $\overline{BC}$ at D. What is BD?



(A) $\frac{\sqrt{3}-1}{2}$ **(B)** $\frac{\sqrt{5}-1}{2}$ **(C)** $\frac{\sqrt{5}+1}{2}$ **(D)** $\frac{\sqrt{6}+\sqrt{2}}{2}$ **(E)** $2\sqrt{3}-1$