11

PROBABILITY

2007A

16. Integers a, b, c, and d, not necessarily distinct, are chosen independently and at random from 0 to 2007, inclusive. What is the probability that $ad - bc$ is even?

$\textbf{(A)}\ \frac{3}{8} \qquad \textbf{(B)}\ \frac{7}{16} \qquad \textbf{(C)}\ \frac{1}{2} \qquad \textbf{(D)}\ \frac{9}{16} \qquad \textbf{(E)}\ \frac{5}{8}$

2008B

16. Two fair coins are to be tossed once. For each head that results, one fair die is to be rolled. What is the probability that the sum of the die rolls is odd? (Note that if no die is rolled, the sum is 0.)

$\textbf{(A)}\ \frac{3}{8} \qquad \textbf{(B)}\ \frac{1}{2} \qquad \textbf{(C)}\ \frac{43}{72} \qquad \textbf{(D)}\ \frac{5}{8} \qquad \textbf{(E)}\ \frac{2}{3}$

2014B

16. Four fair six-sided dice are rolled. What is the probability that at least three of the four dice show the same value?

$\textbf{(A)}\ \frac{1}{36} \qquad \textbf{(B)}\ \frac{7}{72} \qquad \textbf{(C)}\ \frac{1}{9} \qquad \textbf{(D)}\ \frac{5}{36} \qquad \textbf{(E)}\ \frac{1}{6}$

2015B 16. Al, Bill, and Cal will each randomly be assigned a whole number from 1 to 10, inclusive, with no two of them getting the same number. What is the probability that Al's number will be a whole number multiple of Bill's and Bill's number will be a whole number multiple of Cal's?

(A) $\frac{9}{1000}$ **(B)** $\frac{1}{90}$ **(C)** $\frac{1}{80}$ **(D)** $\frac{1}{72}$ **(E)** $\frac{2}{121}$

2006B 17. Bob and Alice each have a bag that contains one ball of each of the colors blue, green, orange, red, and violet. Alice randomly selects one ball from her bag and puts it into Bob's bag. Bob then randomly selects one ball from his bag and puts it into Alice's bag. What is the probability that after this process the contents of the two bags are the same?

(A) $\frac{1}{10}$ **(B)** $\frac{1}{6}$ **(C)** $\frac{1}{5}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{1}{2}$

2008B 17. A poll shows that 70% of all voters approve of the mayor's work. On three separate occasions a pollster selects a voter at random. What is the probability that on exactly one of these three occasions the voter approves of the mayor's work?

(A) 0.063 **(B)** 0.189 **(C)** 0.233 **(D)** 0.333 **(E)** 0.441

2014A 17. Three fair six-sided dice are rolled. What is the probability that the values shown on two of the dice sum to the value shown on the remaining die?

(A) $\frac{1}{6}$ **(B)** $\frac{13}{72}$ **(C)** $\frac{7}{36}$ **(D)** $\frac{5}{24}$ **(E)** $\frac{2}{9}$

2016A 17. Let N be a positive multiple of 5. One red ball and N green balls are arranged in a line in random order. Let $P(N)$ be the probability that at least $\frac{3}{5}$ of the green balls are on the same side of the red ball. Observe that $P(5) = 1$ and that $P(N)$ approaches $\frac{4}{5}$ as N grows large. What is the sum of the digits of the least value of N such that $P(N) < \frac{321}{400}$?

(A) 12 **(B)** 14 **(C)** 16 **(D)** 18 **(E)** 20

2005A

18. Team A and team B play a series. The first team to win three games wins the series. Each team is equally likely to win each game, there are no ties, and the outcomes of the individual games are independent. If team B wins the second game and team A wins the series, what is the probability that team B wins the first game?

(A) $\frac{1}{5}$ (B) $\frac{1}{4}$ (C) $\frac{1}{3}$ (D) $\frac{1}{2}$ (E) $\frac{2}{3}$

2010A

18. Bernardo randomly picks 3 distinct numbers from the set $\{1,2,3,4,5,6,7,8,9\}$ and arranges them in descending order to form a 3-digit number. Silvia randomly picks 3 distinct numbers from the set $\{1,2,3,4,5,6,7,8\}$ and also arranges them in descending order to form a 3-digit number. What is the probability that Bernardo's number is larger than Silvia's number?

(A) $\frac{47}{72}$ (B) $\frac{37}{56}$ (C) $\frac{2}{3}$ (D) $\frac{49}{72}$ (E) $\frac{39}{56}$

2010B

18. Positive integers a, b, and c are randomly and independently selected with replacement from the set $\{1,2,3,\ldots,2010\}$. What is the probability that $abc+ab+a$ is divisible by 3?

(A) $\frac{1}{3}$ (B) $\frac{29}{81}$ (C) $\frac{31}{81}$ (D) $\frac{11}{27}$ (E) $\frac{13}{27}$

2012B

18. Suppose that one of every 500 people in a certain population has a particular disease, which displays no symptoms. A blood test is available for screening for this disease. For a person who has this disease, the test always turns out positive. For a person who does not have the disease, however, there is a 2% false positive rate—in other words, for such people, 98% of the time the test will turn out negative, but 2% of the time the test will turn out positive and will incorrectly indicate that the person has the disease. Let p be the probability that a person who is chosen at random from this population and gets a positive test result actually has the disease. Which of the following is closest to p?

(A) $\frac{1}{98}$ (B) $\frac{1}{9}$ (C) $\frac{1}{11}$ (D) $\frac{49}{99}$ (E) $\frac{98}{99}$

2014B 18. A list of 11 positive integers has a mean of 10, a median of 9, and a unique mode of 8. What is the largest possible value of an integer in the list?

(A) 24 (B) 30 (C) 31 (D) 33 (E) 35

2015B 18. Johann has 64 fair coins. He flips all the coins. Any coin that lands on tails is tossed again. Coins that land on tails on the second toss are tossed a third time. What is the expected number of coins that are now heads?

(A) 32 (B) 40 (C) 48 (D) 56 (E) 64

2017A 18. Amelia has a coin that lands on heads with probability $\frac{1}{3}$, and Blaine has a coin that lands on heads with probability $\frac{2}{5}$. Amelia and Blaine alternately toss their coins until someone gets a head; the first one to get a head wins. All coin tosses are independent. Amelia goes first. The probability that Amelia wins is $\frac{p}{q}$, where p and q are relatively prime positive integers. What is $q-p$?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

2018A 19. A number m is randomly selected from the set $\{11, 13, 15, 17, 19\}$, and a number n is randomly selected from $\{1999, 2000, 2001, \ldots, 2018\}$. What is the probability that m^n has a units digit of 1?

(A) $\frac{1}{5}$ (B) $\frac{1}{4}$ (C) $\frac{3}{10}$ (D) $\frac{7}{20}$ (E) $\frac{2}{5}$

2006A 20. Six distinct positive integers are randomly chosen between 1 and 2006, inclusive. What is the probability that some pair of these integers has a difference that is a multiple of 5?

(A) $\frac{1}{2}$ (B) $\frac{3}{5}$ (C) $\frac{2}{3}$ (D) $\frac{4}{5}$ (E) 1

2008B

20. The faces of a cubical die are marked with the numbers 1, 2, 2, 3, 3, and 4. The faces of a second cubical die are marked with the numbers 1, 3, 4, 5, 6, and 8. Both dice are thrown. What is the probability that the sum of the two top numbers will be 5, 7, or 9?

(A) $\frac{5}{18}$ (B) $\frac{7}{18}$ (C) $\frac{11}{18}$ (D) $\frac{3}{4}$ (E) $\frac{8}{9}$

2012A

20. A 3×3 square is partitioned into 9 unit squares. Each unit square is painted either white or black with each color being equally likely, chosen independently and at random. The square is then rotated 90° clockwise about its center, and every white square in a position formerly occupied by a black square is painted black. The colors of all other squares are left unchanged. What is the probability that the grid is now entirely black?

(A) $\frac{49}{512}$ (B) $\frac{7}{64}$ (C) $\frac{121}{1024}$ (D) $\frac{81}{512}$ (E) $\frac{9}{32}$

2014B

19. Two concentric circles have radii 1 and 2. Two points on the outer circle are chosen independently and uniformly at random. What is the probability that the chord joining the two points intersects the inner circle?

(A) $\frac{1}{6}$ (B) $\frac{1}{4}$ (C) $\frac{2-\sqrt{2}}{2}$ (D) $\frac{1}{3}$ (E) $\frac{1}{2}$

2017B

20. The number $21! = 51{,}090{,}942{,}171{,}709{,}440{,}000$ has over 60,000 positive integer divisors. One of them is chosen at random. What is the probability that it is odd?

(A) $\frac{1}{21}$ (B) $\frac{1}{19}$ (C) $\frac{1}{18}$ (D) $\frac{1}{2}$ (E) $\frac{11}{21}$