

## ALGEBRA WORD PROBLEMS

- 2013A 17. Daphne is visited periodically by her three best friends: Alice, Beatrix, and Claire. Alice visits every third day, Beatrix visits every fourth day, and Claire visits every fifth day. All three friends visited Daphne yesterday. How many days of the next 365-day period will exactly two friends visit her?
- (A) 48      (B) 54      (C) 60      (D) 66      (E) 72
- 2013B 17. Alex has 75 red tokens and 75 blue tokens. There is a booth where Alex can give two red tokens and receive in return a silver token and a blue token, and another booth where Alex can give three blue tokens and receive in return a silver token and a red token. Alex continues to exchange tokens until no more exchanges are possible. How many silver tokens will Alex have at the end?
- (A) 62      (B) 82      (C) 83      (D) 102      (E) 103
- 2017B 17. Call a positive integer *monotonous* if it is a one-digit number or its digits, when read from left to right, form either a strictly increasing or a strictly decreasing sequence. For example, 3, 23578, and 987620 are *monotonous*, but 88, 7434, and 23557 are not. How many *monotonous* positive integers are there?
- (A) 1024      (B) 1524      (C) 1533      (D) 1536      (E) 2048

- 2015A 18. Hexadecimal (base-16) numbers are written using the numeric digits 0 through 9 as well as the letters  $A$  through  $F$  to represent 10 through 15. Among the first 1000 positive integers, there are  $n$  whose hexadecimal representation contains only numeric digits. What is the sum of the digits of  $n$ ?
- (A) 17      (B) 18      (C) 19      (D) 20      (E) 21
- 2013A 19. In base 10, the number 2013 ends in the digit 3. In base 9, on the other hand, the same number is written as  $(2676)_9$  and ends in the digit 6. For how many positive integers  $b$  does the base- $b$  representation of 2013 end in the digit 3?
- (A) 6      (B) 9      (C) 13      (D) 16      (E) 18
- 2003A 20. A base-10 three-digit number  $n$  is selected at random. Which of the following is closest to the probability that the base-9 representation and the base-11 representation of  $n$  are both three-digit numerals?
- (A) 0.3      (B) 0.4      (C) 0.5      (D) 0.6      (E) 0.7
- 2012B 20. Bernardo and Silvia play the following game. An integer between 0 and 999, inclusive, is selected and given to Bernardo. Whenever Bernardo receives a number, he doubles it and passes the result to Silvia. Whenever Silvia receives a number, she adds 50 to it and passes the result to Bernardo. The winner is the last person who produces a number less than 1000. Let  $N$  be the smallest initial number that results in a win for Bernardo. What is the sum of the digits of  $N$ ?
- (A) 7      (B) 8      (C) 9      (D) 10      (E) 11

- 2016A 20. For some particular value of  $N$ , when  $(a + b + c + d + 1)^N$  is expanded and like terms are combined, the resulting expression contains exactly 1001 terms that include all four variables,  $a$ ,  $b$ ,  $c$ , and  $d$ , each to some positive power. What is  $N$ ?
- (A) 9      (B) 14      (C) 16      (D) 17      (E) 19