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SEQUENCE AND SERIES

2016B 16. The sum of an infinite geometric series is a positive number S, and the second term in the series is 1. What is the smallest possible value of S?

(A) $\frac{1+\sqrt{5}}{2}$ **(B)** 2 **(C)** $\sqrt{5}$ **(D)** 3 **(E)** 4

2018B 16. Let $a_1, a_2, \ldots, a_{2018}$ be a strictly increasing sequence of positive integers such that

$$a_1 + a_2 + \cdots + a_{2018} = 2018^{2018}.$$

What is the remainder when $a_1^3 + a_2^3 + \cdots + a_{2018}^3$ is divided by 6?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

2004A 18. A sequence of three real numbers forms an arithmetic progression with a first term of 9. If 2 is added to the second term and 20 is added to the third term, the three resulting numbers form a geometric progression. What is the smallest possible value for the third term of the geometric progression?

(A) 1 **(B)** 4 **(C)** 36 **(D)** 49 **(E)** 81

2006B 18. Let $a_1, a_2, \ldots$ be a sequence for which

$$a_1 = 2, \quad a_2 = 3, \quad \text{and} \quad a_n = \frac{a_{n-1}}{a_{n-2}} \text{ for each positive integer } n \geq 3.$$

What is a_{2006}?

(A) $\frac{1}{2}$ **(B)** $\frac{2}{3}$ **(C)** $\frac{3}{2}$ **(D)** 2 **(E)** 3

2002B 19. Suppose that $\{a_n\}$ is an arithmetic sequence with

$$a_1 + a_2 + \cdots + a_{100} = 100 \quad \text{and} \quad a_{101} + a_{102} + \cdots + a_{200} = 200.$$

What is the value of $a_2 - a_1$?

(A) 0.0001 **(B)** 0.001 **(C)** 0.01 **(D)** 0.1 **(E)** 1

2004B 19. In the sequence 2001, 2002, 2003, ..., each term after the third is found by subtracting the previous term from the sum of the two terms that precede that term. For example, the fourth term is $2001 + 2002 - 2003 = 2000$. What is the 2004^{th} term in this sequence?

(A) -2004 **(B)** -2 **(C)** 0 **(D)** 4003 **(E)** 6007

2006A 19. How many non-similar triangles have angles whose degree measures are distinct positive integers in arithmetic progression?

(A) 0 **(B)** 1 **(C)** 59 **(D)** 89 **(E)** 178

2017A 20. Let $S(n)$ equal the sum of the digits of positive integer n. For example, $S(1507) = 13$. For a particular positive integer n, $S(n) = 1274$. Which of the following could be the value of $S(n+1)$?

(A) 1 **(B)** 3 **(C)** 12 **(D)** 1239 **(E)** 1265