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2D WORD PROBLEMS

2008A 16. Points A and B lie on a circle centered at O, and $\angle AOB = 60°$. A second circle is internally tangent to the first and tangent to both $\overline{OA}$ and $\overline{OB}$. What is the ratio of the area of the smaller circle to that of the larger circle?

(A) $\frac{1}{16}$ **(B)** $\frac{1}{9}$ **(C)** $\frac{1}{8}$ **(D)** $\frac{1}{6}$ **(E)** $\frac{1}{4}$

2010B 16. A square of side length 1 and a circle of radius $\sqrt{3}/3$ share the same center. What is the area inside the circle, but outside the square?

(A) $\frac{\pi}{3} - 1$ **(B)** $\frac{2\pi}{9} - \frac{\sqrt{3}}{3}$ **(C)** $\frac{\pi}{18}$ **(D)** $\frac{1}{4}$ **(E)** $2\pi/9$

2003A 17. The number of inches in the perimeter of an equilateral triangle equals the number of square inches in the area of its circumscribed circle. What is the radius, in inches, of the circle?

(A) $\frac{3\sqrt{2}}{\pi}$ **(B)** $\frac{3\sqrt{3}}{\pi}$ **(C)** $\sqrt{3}$ **(D)** $\frac{6}{\pi}$ **(E)** $\sqrt{3}\pi$

2007B

17. Point P is inside equilateral $\triangle ABC$. Points Q, R, and S are the feet of the perpendiculars from P to $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$, respectively. Given that $PQ = 1$, $PR = 2$, and $PS = 3$, what is AB?

(A) 4 **(B)** $3\sqrt{3}$ **(C)** 6 **(D)** $4\sqrt{3}$ **(E)** 9

2018B

17. In rectangle $PQRS$, $PQ = 8$ and $QR = 6$. Points A and B lie on $\overline{PQ}$, points C and D lie on $\overline{QR}$, points E and F lie on $\overline{RS}$, and points G and H lie on $\overline{SP}$ so that $AP = BQ < 4$ and the convex octagon $ABCDEFGH$ is equilateral. The length of a side of this octagon can be expressed in the form $k + m\sqrt{n}$, where k, m, and n are integers and n is not divisible by the square of any prime. What is $k+m+n$?

(A) 1 **(B)** 7 **(C)** 21 **(D)** 92 **(E)** 106

2000

18. Charlyn walks completely around the boundary of a square whose sides are each 5 km long. From any point on her path she can see exactly 1 km horizontally in all directions. What is the area of the region consisting of all points Charlyn can see during her walk, expressed in square kilometers and rounded to the nearest whole number?

(A) 24 **(B)** 27 **(C)** 39 **(D)** 40 **(E)** 42

2002B

18. Four distinct circles are drawn in a plane. What is the maximum number of points where at least two of the circles intersect?

(A) 8 **(B)** 9 **(C)** 10 **(D)** 12 **(E)** 16

2011B

18. Rectangle $ABCD$ has $AB = 6$ and $BC = 3$. Point M is chosen on side AB so that $\angle AMD = \angle CMD$. What is the degree measure of $\angle AMD$?

(A) 15 **(B)** 30 **(C)** 45 **(D)** 60 **(E)** 75

2013A 18. Let points $A = (0,0)$, $B = (1,2)$, $C = (3,3)$, and $D = (4,0)$. Quadrilateral $ABCD$ is cut into equal area pieces by a line passing through A. This line intersects $\overline{CD}$ at point $\left(\frac{p}{q}, \frac{r}{s}\right)$, where these fractions are in lowest terms. What is $p + q + r + s$?

(A) 54 **(B)** 58 **(C)** 62 **(D)** 70 **(E)** 75

2014A 18. A square in the coordinate plane has vertices whose y-coordinates are 0, 1, 4, and 5. What is the area of the square?

(A) 16 **(B)** 17 **(C)** 25 **(D)** 26 **(E)** 27

2002A 19. Spot's doghouse has a regular hexagonal base that measures one yard on each side. He is tethered to a vertex with a two-yard rope. What is the area, in square yards, of the region outside the doghouse that Spot can reach?

(A) $\frac{2}{3}\pi$ **(B)** 2π **(C)** $\frac{5}{2}\pi$ **(D)** $\frac{8}{3}\pi$ **(E)** 3π

2008A 19. Rectangle $PQRS$ lies in a plane with $PQ = RS = 2$ and $QR = SP = 6$. The rectangle is rotated 90° clockwise about R, then rotated 90° clockwise about the point that S moved to after the first rotation. What is the length of the path traveled by point P?

(A) $\left(2\sqrt{3} + \sqrt{5}\right)\pi$ **(B)** 6π **(C)** $\left(3 + \sqrt{10}\right)\pi$ **(D)** $\left(\sqrt{3} + 2\sqrt{5}\right)\pi$

(E) $2\sqrt{10}\pi$

2010A 19. Equiangular hexagon $ABCDEF$ has side lengths $AB = CD = EF = 1$ and $BC = DE = FA = r$. The area of $\triangle ACE$ is 70% of the area of the hexagon. What is the sum of all possible values of r?

(A) $\frac{4\sqrt{3}}{3}$ **(B)** $\frac{10}{3}$ **(C)** 4 **(D)** $\frac{17}{4}$ **(E)** 6

2012B

19. In rectangle $ABCD$, $AB = 6$, $AD = 30$, and G is the midpoint of $\overline{AD}$. Segment AB is extended 2 units beyond B to point E, and F is the intersection of $\overline{ED}$ and $\overline{BC}$. What is the area of $BFDG$?

(A) $\frac{133}{2}$ **(B)** 67 **(C)** $\frac{135}{2}$ **(D)** 68 **(E)** $\frac{137}{2}$

2001

20. A regular octagon is formed by cutting an isosceles right triangle from each of the corners of a square with sides of length 2000. What is the length of each side of the octagon?

(A) $\frac{1}{3}(2000)$ **(B)** $2000(\sqrt{2}-1)$ **(C)** $2000(2-\sqrt{2})$

(D) 1000 **(E)** $1000\sqrt{2}$

2005A

20. An equiangular octagon has four sides of length 1 and four sides of length $\sqrt{2}/2$, arranged so that no two consecutive sides have the same length. What is the area of the octagon?

(A) $\frac{7}{2}$ **(B)** $\frac{7\sqrt{2}}{2}$ **(C)** $\frac{5+4\sqrt{2}}{2}$ **(D)** $\frac{4+5\sqrt{2}}{2}$ **(E)** 7

2006B

20. In rectangle $ABCD$, we have $A = (6, -22)$, $B = (2006, 178)$, and $D = (8, y)$, for some integer y. What is the area of rectangle $ABCD$?

(A) 4000 **(B)** 4040 **(C)** 4400 **(D)** 40,000 **(E)** 40,400

2008A

20. Trapezoid $ABCD$ has bases $\overline{AB}$ and $\overline{CD}$ and diagonals intersecting at K. Suppose that $AB = 9$, $DC = 12$, and the area of $\triangle AKD$ is 24. What is the area of trapezoid $ABCD$?

(A) 92 **(B)** 94 **(C)** 96 **(D)** 98 **(E)** 100

2011A

20. Two points on the circumference of a circle of radius r are selected independently and at random. From each point a chord of length r is drawn in a clockwise direction. What is the probability that the two chords intersect?

(A) $\frac{1}{6}$ **(B)** $\frac{1}{5}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{1}{2}$

2011B

20. Rhombus $ABCD$ has side length 2 and $\angle B = 120°$. Region R consists of all points inside the rhombus that are closer to vertex B than any of the other three vertices. What is the area of R?

(A) $\frac{\sqrt{3}}{3}$ **(B)** $\frac{\sqrt{3}}{2}$ **(C)** $\frac{2\sqrt{3}}{3}$ **(D)** $1+\frac{\sqrt{3}}{3}$ **(E)** 2

2013A

20. A unit square is rotated $45°$ about its center. What is the area of the region swept out by the interior of the square?

(A) $1-\frac{\sqrt{2}}{2}+\frac{\pi}{4}$ **(B)** $\frac{1}{2}+\frac{\pi}{4}$ **(C)** $2-\sqrt{2}+\frac{\pi}{4}$

(D) $\frac{\sqrt{2}}{2}+\frac{\pi}{4}$ **(E)** $1+\frac{\sqrt{2}}{4}+\frac{\pi}{8}$

2016B

20. A dilation of the plane—that is, a size transformation with a positive scale factor—sends the circle of radius 2 centered at $A(2,2)$ to the circle of radius 3 centered at $A'(5,6)$. What distance does the origin $O(0,0)$ move under this transformation?

(A) 0 **(B)** 3 **(C)** $\sqrt{13}$ **(D)** 4 **(E)** 5