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TRIANGLES

2009B 16. Points A and C lie on a circle centered at O, each of $\overline{BA}$ and $\overline{BC}$ are tangent to the circle, and $\triangle ABC$ is equilateral. The circle intersects $\overline{BO}$ at D. What is $\frac{BD}{BO}$?

(A) $\dfrac{\sqrt{2}}{3}$ (B) $\dfrac{1}{2}$ (C) $\dfrac{\sqrt{3}}{3}$ (D) $\dfrac{\sqrt{2}}{2}$ (E) $\dfrac{\sqrt{3}}{2}$

2010A 16. Nondegenerate $\triangle ABC$ has integer side lengths, $\overline{BD}$ is an angle bisector, $AD = 3$, and $DC = 8$. What is the smallest possible value of the perimeter?

(A) 30   (B) 33   (C) 35   (D) 36   (E) 37

2013A 16. A triangle with vertices $(6,5)$, $(8,-3)$, and $(9,1)$ is reflected about the line $x = 8$ to create a second triangle. What is the area of the union of the two triangles?

(A) 9   (B) $\dfrac{28}{3}$ (C) 10   (D) $\dfrac{31}{3}$ (E) $\dfrac{32}{3}$

2016A

16. A triangle with vertices $A(0, 2)$, $B(-3, 2)$, and $C(-3, 0)$ is reflected about the x-axis; then the image $\triangle A'B'C'$ is rotated counterclockwise around the origin by $90°$ to produce $\triangle A''B''C''$. Which of the following transformations will return $\triangle A''B''C''$ to $\triangle ABC$?

(A) counterclockwise rotation around the origin by $90°$

(B) clockwise rotation around the origin by $90°$

(C) reflection about the x-axis

(D) reflection about the line $y = x$

(E) reflection about the y-axis

2018A

16. Right triangle ABC has leg lengths $AB = 20$ and $BC = 21$. Including $\overline{AB}$ and $\overline{BC}$, how many line segments with integer length can be drawn from vertex B to a point on hypotenuse $\overline{AC}$?

(A) 5   (B) 8   (C) 12   (D) 13   (E) 15

2002B

17. A regular octagon $ABCDEFGH$ has sides of length two. Find the area of $\triangle ADG$.

(A) $4 + 2\sqrt{2}$ (B) $6 + \sqrt{2}$ (C) $4 + 3\sqrt{2}$ (D) $3 + 4\sqrt{2}$ (E) $8 + \sqrt{2}$

2008A

17. An equilateral triangle has side length 6. What is the area of the region containing all points that are outside the triangle and not more than 3 units from a point of the triangle?

(A) $36 + 24\sqrt{3}$ (B) $54 + 9\pi$ (C) $54 + 18\sqrt{3} + 6\pi$ (D) $\left(2\sqrt{3} + 3\right)^2 \pi$

(E) $9\left(\sqrt{3} + 1\right)^2 \pi$

2015A 17. A line that passes through the origin intersects both the line $x = 1$ and the line $y = 1 + \frac{\sqrt{3}}{3}x$. The three lines create an equilateral triangle. What is the perimeter of the triangle?

(A) $2\sqrt{6}$ (B) $2 + 2\sqrt{3}$ (C) 6   (D) $3 + 2\sqrt{3}$ (E) $6 + \frac{\sqrt{3}}{3}$

2008A 18. A right triangle has perimeter 32 and area 20. What is the length of its hypotenuse?

(A) $\dfrac{57}{4}$ (B) $\dfrac{59}{4}$ (C) $\dfrac{61}{4}$ (D) $\dfrac{63}{4}$ (E) $\dfrac{65}{4}$

2009B 18. Rectangle $ABCD$ has $AB = 8$ and $BC = 6$. Point M is the midpoint of diagonal $\overline{AC}$, and E is on $\overline{AB}$ with $\overline{ME} \perp \overline{AC}$. What is the area of $\triangle AME$?

(A) $\dfrac{65}{8}$ (B) $\dfrac{25}{3}$ (C) 9   (D) $\dfrac{75}{8}$ (E) $\dfrac{85}{8}$

2010B 19. A circle with center O has area 156π. Triangle ABC is equilateral, $\overline{BC}$ is a chord on the circle, $OA = 4\sqrt{3}$, and point O is outside $\triangle ABC$. What is the side length of $\triangle ABC$?

(A) $2\sqrt{3}$ (B) 6   (C) $4\sqrt{3}$ (D) 12   (E) 18

2015A 19. The isosceles right triangle ABC has right angle at C and area 12.5. The rays trisecting $\angle ACB$ intersect AB at D and E. What is the area of $\triangle CDE$?

(A) $\dfrac{5\sqrt{2}}{3}$ (B) $\dfrac{50\sqrt{3} - 75}{4}$ (C) $\dfrac{15\sqrt{3}}{8}$ (D) $\dfrac{50 - 25\sqrt{3}}{2}$ (E) $\dfrac{25}{6}$

2015B

19. In $\triangle ABC$, $\angle C = 90°$ and $AB = 12$. Squares $ABXY$ and $ACWZ$ are constructed outside of the triangle. The points X, Y, Z, and W lie on a circle. What is the perimeter of the triangle?

(A) $12 + 9\sqrt{3}$ (B) $18 + 6\sqrt{3}$ (C) $12 + 12\sqrt{2}$ (D) 30 (E) 32