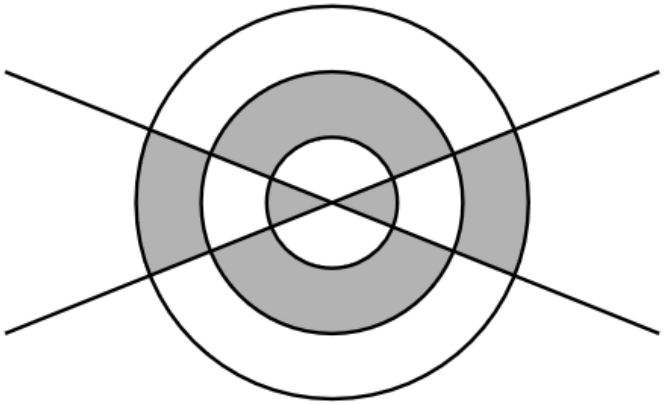
1

2D GEOMETRY

2004A

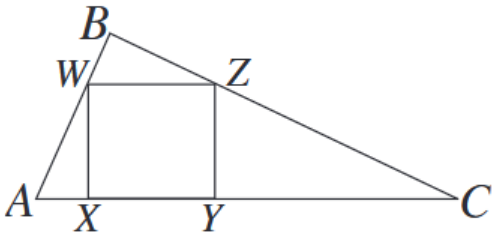
21. Two distinct lines pass through the center of three concentric circles of radii 3, 2, and 1. The area of the shaded region in the diagram is 8/13 of the area of the unshaded region. What is the radian measure of the acute angle formed by the two lines? (Note: π radians is 180 degrees.)



(A) $\frac{\pi}{8}$ **(B)** $\frac{\pi}{7}$ **(C)** $\frac{\pi}{6}$ **(D)** $\frac{\pi}{5}$ **(E)** $\frac{\pi}{4}$

2007B

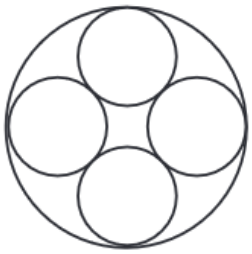
21. Right $\triangle ABC$ has $AB = 3$, $BC = 4$, and $AC = 5$. Square $XYZW$ is inscribed in $\triangle ABC$ with X and Y on $\overline{AC}$, W on $\overline{AB}$, and Z on $\overline{BC}$. What is the side length of the square?



(A) $\frac{3}{2}$ **(B)** $\frac{60}{37}$ **(C)** $\frac{12}{7}$ **(D)** $\frac{23}{13}$ **(E)** 2

2009A

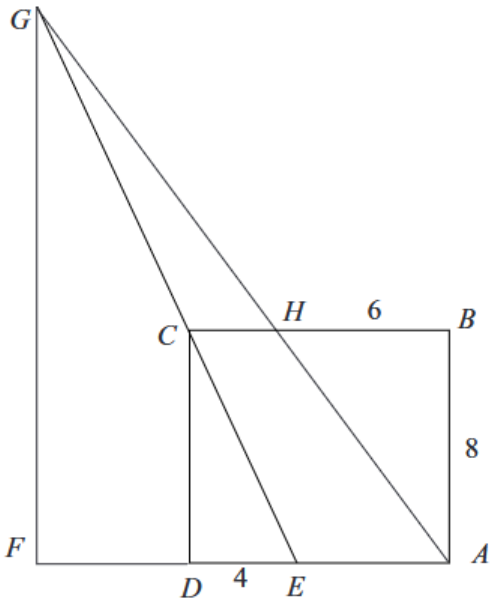
21. Many Gothic cathedrals have windows with portions containing a ring of congruent circles that are circumscribed by a larger circle. In the figure shown, the number of smaller circles is four. What is the ratio of the sum of the areas of the four smaller circles to the area of the larger circle?



(A) $3-2\sqrt{2}$ **(B)** $2-\sqrt{2}$ **(C)** $4(3-2\sqrt{2})$ **(D)** $\frac{1}{2}(3-\sqrt{2})$ **(E)** $2\sqrt{2}-2$

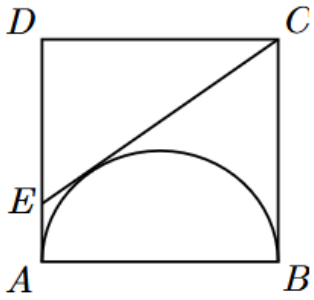
2003A

22. In rectangle $ABCD$, we have $AB=8$, $BC=9$, H is on $\overline{BC}$ with $BH=6$, E is on AD with $DE=4$, line EC intersects line AH at G, and F is on line AD with $\overline{GF}\perp\overline{AF}$. Find the length $\overline{GF}$.



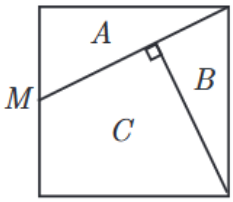
(A) 16 **(B)** 20 **(C)** 24 **(D)** 28 **(E)** 30

2004A 22. Square $ABCD$ has side length 2. A semicircle with diameter $\overline{AB}$ is constructed inside the square, and the tangent to the semicircle from C intersects side $\overline{AD}$ at E. What is the length of $\overline{CE}$?



(A) $\frac{2+\sqrt{5}}{2}$ **(B)** $\sqrt{5}$ **(C)** $\sqrt{6}$ **(D)** $\frac{5}{2}$ **(E)** $5-\sqrt{5}$

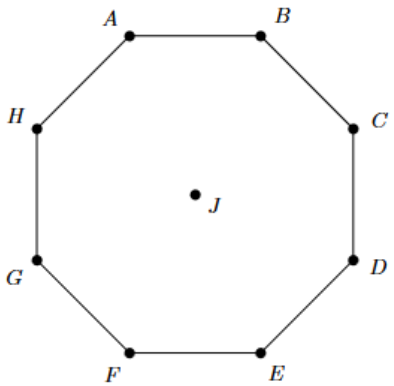
2009B 22. A cubical cake with edge length 2 inches is iced on the sides and the top. It is cut vertically into three pieces as shown in this top view, where M is the midpoint of a top edge. The piece whose top is triangle B contains c cubic inches of cake and s square inches of icing. What is $c+s$?



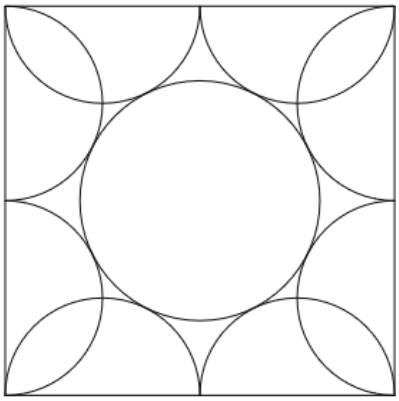
(A) $\frac{24}{5}$ **(B)** $\frac{32}{5}$ **(C)** $8+\sqrt{5}$ **(D)** $5+\frac{16\sqrt{5}}{5}$ **(E)** $10+5\sqrt{5}$

2013B 22. The regular octagon $ABCDEFGH$ has its center at J. Each of the vertices and the center are to be associated with one of the digits 1 through 9, with each digit used once, in such a way that the sums of the numbers on the lines AJE, BJF, CJG, and DJH are equal. In how many ways can this be done?

(A) 384 **(B)** 576 **(C)** 1152 **(D)** 1680 **(E)** 3456

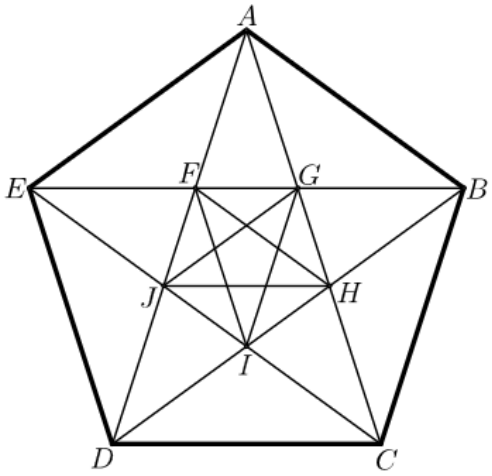


2014B 22. Eight semicircles line the inside of a square with side length 2 as shown. What is the radius of the circle tangent to all of these semicircles?



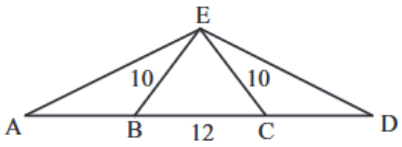
(A) $\frac{1+\sqrt{2}}{4}$ (B) $\frac{\sqrt{5}-1}{2}$ (C) $\frac{\sqrt{3}+1}{4}$ (D) $\frac{2\sqrt{3}}{5}$ (E) $\frac{\sqrt{5}}{3}$

2015B 22. In the figure shown below, $ABCDE$ is a regular pentagon and $AG = 1$. What is $FG + JH + CD$?



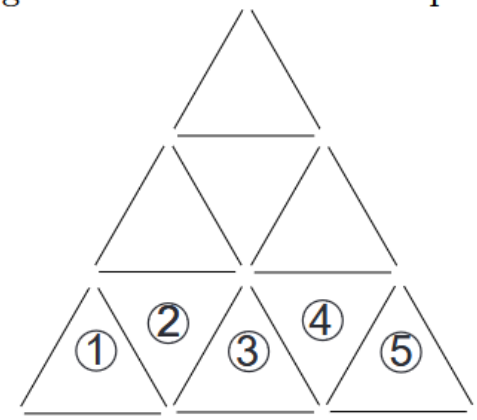
(A) 3 (B) $12 - 4\sqrt{5}$ (C) $\frac{5+2\sqrt{5}}{3}$ (D) $1 + \sqrt{5}$ (E) $\frac{11+11\sqrt{5}}{10}$

2002A 23. Points A, B, C, and D lie on a line, in that order, with $AB = CD$ and $BC = 12$. Point E is not on the line, and $BE = CE = 10$. The perimeter of $\triangle AED$ is twice the perimeter of $\triangle BEC$. Find AB.



(A) 15/2 **(B)** 8 **(C)** 17/2 **(D)** 9 **(E)** 19/2

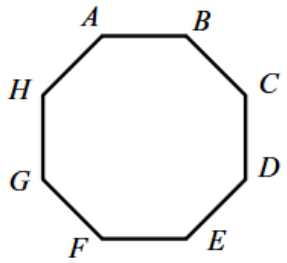
2003A 23. A large equilateral triangle is constructed by using toothpicks to create rows of small equilateral triangles. For example, in the figure we have 3 rows of small congruent equilateral triangles, with 5 small triangles in the base row. How many toothpicks would be needed to construct a large equilateral triangle if the base row of the triangle consists of 2003 small equilateral triangles?



(A) 1,004,004 **(B)** 1,005,006 **(C)** 1,507,509 **(D)** 3,015,018

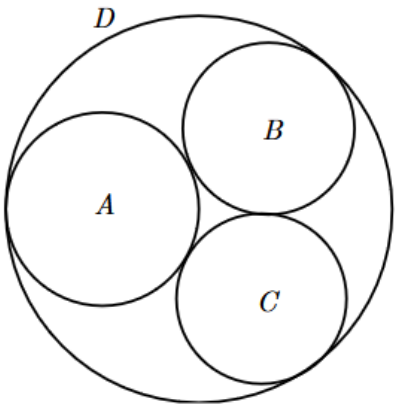
(E) 6,021,018

2003B 23. A regular octagon $ABCDEFGH$ has an area of one square unit. What is the area of the rectangle $ABEF$?



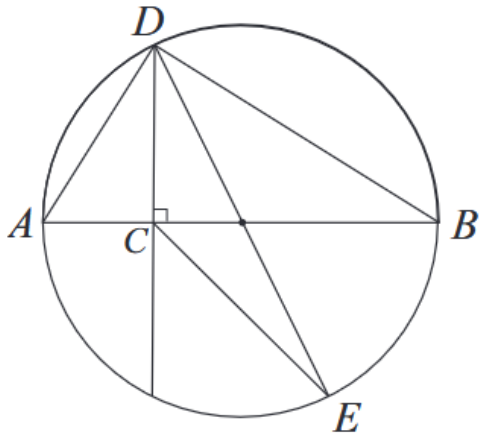
(A) $1-\frac{\sqrt{2}}{2}$ **(B)** $\frac{\sqrt{2}}{4}$ **(C)** $\sqrt{2}-1$ **(D)** $\frac{1}{2}$ **(E)** $\frac{1+\sqrt{2}}{4}$

2004A 23. Circles A, B, and C are externally tangent to each other and internally tangent to circle D. Circles B and C are congruent. Circle A has radius 1 and passes through the center of D. What is the radius of circle B?



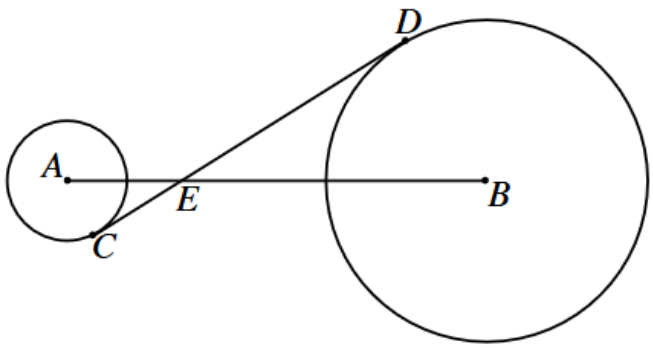
(A) $\frac{2}{3}$ **(B)** $\frac{\sqrt{3}}{2}$ **(C)** $\frac{7}{8}$ **(D)** $\frac{8}{9}$ **(E)** $\frac{1+\sqrt{3}}{3}$

2005A 23. Let $\overline{AB}$ be a diameter of a circle and C be a point on $\overline{AB}$ with $2 \cdot AC = BC$. Let D and E be points on the circle such that $\overline{DC} \perp \overline{AB}$ and $\overline{DE}$ is a second diameter. What is the ratio of the area of $\triangle DCE$ to the area of $\triangle ABD$?



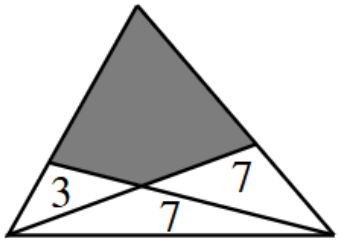
(A) $\frac{1}{6}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{1}{3}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{2}{3}$

2006A 23. Circles with centers A and B have radii 3 and 8, respectively. A common internal tangent touches the circles at C and D, as shown. Lines AB and CD intersect at E, and $AE = 5$. What is CD ?



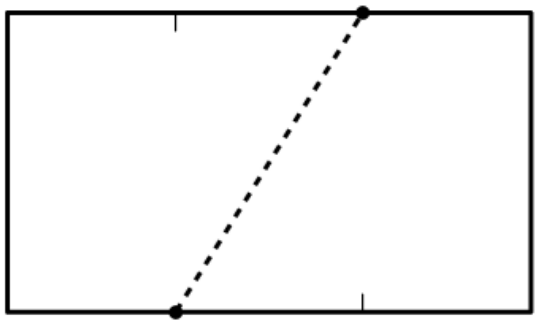
(A) 13 **(B)** $\frac{44}{3}$ **(C)** $\sqrt{221}$ **(D)** $\sqrt{255}$ **(E)** $\frac{55}{3}$

2006B 23. A triangle is partitioned into three triangles and a quadrilateral by drawing two lines from vertices to their opposite sides. The areas of the three triangles are 3, 7, and 7, as shown. What is the area of the shaded quadrilateral?



(A) 15 **(B)** 17 **(C)** $\frac{35}{2}$ **(D)** 18 **(E)** $\frac{55}{3}$

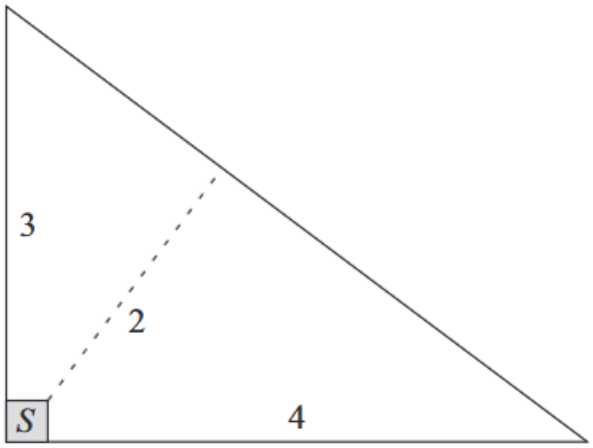
2014A 23. A rectangular piece of paper whose length is $\sqrt{3}$ times the width has area A. The paper is divided into three equal sections along the opposite lengths, and then a dotted line is drawn from the first divider to the second divider on the opposite side as shown. The paper is then folded flat along this dotted line to create a new shape with area B. What is the ratio $B : A$?



(A) 1 : 2 **(B)** 3 : 5 **(C)** 2 : 3 **(D)** 3 : 4 **(E)** 4 : 5

2018A

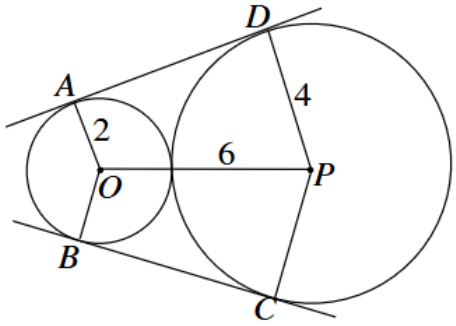
23. Farmer Pythagoras has a field in the shape of a right triangle. The right triangle's legs have lengths of 3 and 4 units. In the corner where those sides meet at a right angle, he leaves a small unplanted square S so that from the air it looks like the right angle symbol. The rest of the field is planted. The shortest distance from S to the hypotenuse is 2 units. What fraction of the field is planted?



(A) $\frac{25}{27}$ **(B)** $\frac{26}{27}$ **(C)** $\frac{73}{75}$ **(D)** $\frac{145}{147}$ **(E)** $\frac{74}{75}$

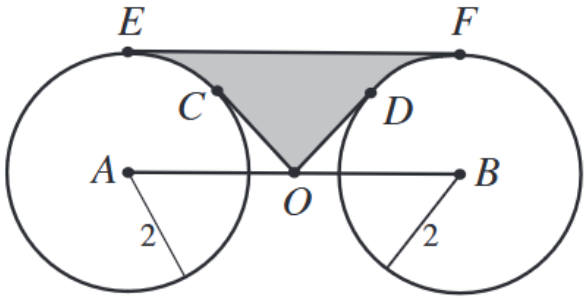
2006B

24. Circles with centers at O and P have radii 2 and 4, respectively, and are externally tangent. Points A and B on the circle with center O and points C and D on the circle with center P are such that $\overline{AD}$ and $\overline{BC}$ are common external tangents to the circles. What is the area of the concave hexagon $AOBCPD$?



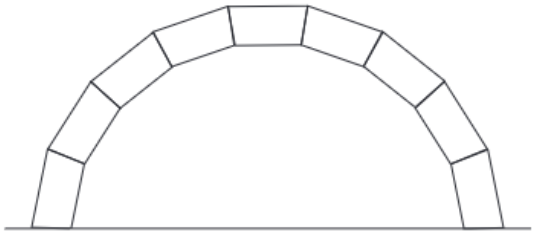
(A) $18\sqrt{3}$ **(B)** $24\sqrt{2}$ **(C)** 36 **(D)** $24\sqrt{3}$ **(E)** $32\sqrt{2}$

2007A 24. Circles centered at A and B each have radius 2, as shown. Point O is the midpoint of $\overline{AB}$, and $OA = 2\sqrt{2}$. Segments OC and OD are tangent to the circles centered at A and B, respectively, and $\overline{EF}$ is a common tangent. What is the area of the shaded region $ECODF$?



(A) $\frac{8\sqrt{2}}{3}$ **(B)** $8\sqrt{2}-4-\pi$ **(C)** $4\sqrt{2}$ **(D)** $4\sqrt{2}+\frac{\pi}{8}$ **(E)** $8\sqrt{2}-2-\frac{\pi}{2}$

2009B 24. The keystone arch is an ancient architectural feature. It is composed of congruent isosceles trapezoids fitted together along the non-parallel sides, as shown. The bottom sides of the two end trapezoids are horizontal. In an arch made with 9 trapezoids, let x be the angle measure in degrees of the larger interior angle of the trapezoid. What is x?

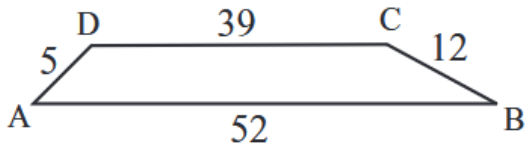


(A) 100 **(B)** 102 **(C)** 104 **(D)** 106 **(E)** 108

2002A

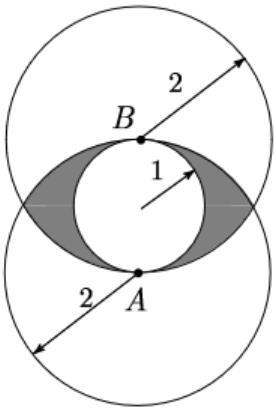
25. In trapezoid $ABCD$ with bases $\overline{AB}$ and $\overline{CD}$, we have $AB = 52$, $BC = 12$, $CD = 39$, and $DA = 5$. The area of $ABCD$ is

(A) 182 **(B)** 195 **(C)** 210 **(D)** 234 **(E)** 260



2004B

25. A circle of radius 1 is internally tangent to two circles of radius 2 at points A and B, where AB is a diameter of the smaller circle. What is the area of the region, shaded in the figure, that is outside the smaller circle and inside each of the two larger circles?

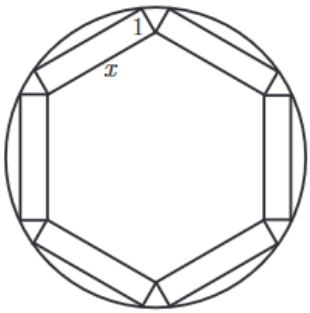


(A) $\frac{5}{3}\pi - 3\sqrt{2}$ **(B)** $\frac{5}{3}\pi - 2\sqrt{3}$ **(C)** $\frac{8}{3}\pi - 3\sqrt{3}$ **(D)** $\frac{8}{3}\pi - 3\sqrt{2}$

(E) $\frac{8}{3}\pi - 2\sqrt{3}$

2008A

25. A round table has radius 4. Six rectangular place mats are placed on the table. Each place mat has width 1 and length x as shown. They are positioned so that each mat has two corners on the edge of the table, these two corners being end points of the same side of length x. Further, the mats are positioned so that the inner corners each touch an inner corner of an adjacent mat. What is x?



(A) $2\sqrt{5}-\sqrt{3}$ **(B)** 3 **(C)** $\frac{3\sqrt{7}-\sqrt{3}}{2}$ **(D)** $2\sqrt{3}$ **(E)** $\frac{5+2\sqrt{3}}{2}$