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DIVISOR/GCF

2009B

21. What is the remainder when $3^0 + 3^1 + 3^2 + \cdots + 3^{2009}$ is divided by 8?

(A) 0   (B) 1   (C) 2   (D) 4   (E) 6

2005B

22. For how many positive integers n less than or equal to 24 is $n!$ evenly divisible by $1 + 2 + \cdots + n$?

(A) 8   (B) 12   (C) 16   (D) 17   (E) 21

2016A

22. For some positive integer n, the number $110n^3$ has 110 positive integer divisors, including 1 and the number $110n^3$. How many positive integer divisors does the number $81n^4$ have?

(A) 110   (B) 191   (C) 261   (D) 325   (E) 425

2018A

22. Let a, b, c, and d be positive integers such that $\gcd(a,b) = 24$, $\gcd(b,c) = 36$, $\gcd(c,d) = 54$, and $70 < \gcd(d,a) < 100$. Which of the following must be a divisor of a?

(A) 5   (B) 7   (C) 11   (D) 13   (E) 17

2011B

23. What is the hundreds digit of 2011^{2011} ?

(A) 1   (B) 4   (C) 5   (D) 6   (E) 9

2017B

23. Let $N = 123456789101112\ldots4344$ be the 79-digit number that is formed by writing the integers from 1 to 44 in order, one after the other. What is the remainder when N is divided by 45 ?

(A) 1   (B) 4   (C) 9   (D) 18   (E) 44

2018B

23. How many ordered pairs (a, b) of positive integers satisfy the equation

$$a \cdot b + 63 = 20 \cdot \text{lcm}(a, b) + 12 \cdot \gcd(a, b),$$

where $\gcd(a, b)$ denotes the greatest common divisor of a and b, and $\text{lcm}(a, b)$ denotes their least common multiple?

(A) 0   (B) 2   (C) 4   (D) 6   (E) 8

2008A

24. Let $k = 2008^2 + 2^{2008}$. What is the units digit of $k^2 + 2^k$?

(A) 0   (B) 2   (C) 4   (D) 6   (E) 8

2010A

24. The number obtained from the last two nonzero digits of 90! is equal to n. What is n?

(A) 12   (B) 32   (C) 48   (D) 52   (E) 68

2013B

24. A positive integer n is *nice* if there is a positive integer m with exactly four positive divisors (including 1 and m) such that the sum of the four divisors is equal to n. How many numbers in the set $\{2010, 2011, 2012, \ldots, 2019\}$ are nice?

(A) 1   (B) 2   (C) 3   (D) 4   (E) 5

2014A

24. A sequence of natural numbers is constructed by listing the first 4, then skipping one, listing the next 5, skipping 2, listing 6, skipping 3, and, on the nth iteration, listing $n+3$ and skipping n. The sequence begins $1, 2, 3, 4, 6, 7, 8, 9, 10, 13$. What is the 500,000th number in the sequence?

(A) 996,506   (B) 996,507   (C) 996,508   (D) 996,509   (E) 996,510

2001

25. How many positive integers not exceeding 2001 are multiples of 3 or 4 but not 5?

(A) 768   (B) 801   (C) 934   (D) 1067   (E) 1167

2003B

25. How many distinct four-digit numbers are divisible by 3 and have 23 as their last two digits?

(A) 27   (B) 30   (C) 33   (D) 81   (E) 90

2007B 25. How many pairs of positive integers (a, b) are there such that a and b have no common factors greater than 1 and

$$\frac{a}{b} + \frac{14b}{9a}$$

is an integer?

(A) 4   (B) 6   (C) 9   (D) 12   (E) infinitely many

2016A 25. How many ordered triples (x, y, z) of positive integers satisfy $\text{lcm}(x, y) = 72$, $\text{lcm}(x, z) = 600$, and $\text{lcm}(y, z) = 900$?

(A) 15   (B) 16   (C) 24   (D) 27   (E) 64