16

SEQUENCE AND SERIES

2004B 21. Let $1, 4, \ldots$ and $9, 16, \ldots$ be two arithmetic progressions. The set S is the union of the first 2004 terms of each sequence. How many distinct numbers are in S?

(A) 3722 **(B)** 3732 **(C)** 3914 **(D)** 3924 **(E)** 4007

2010B

24. A high school basketball game between the Raiders and the Wildcats was tied at the end of the first quarter. The number of points scored by the Raiders in each of the four quarters formed an increasing geometric sequence, and the number of points scored by the Wildcats in each of the four quarters formed an increasing arithmetic sequence. At the end of the fourth quarter, the Raiders had won by one point. Neither team scored more than 100 points. What was the total number of points scored by the two teams in the first half?

(A) 30 **(B)** 31 **(C)** 32 **(D)** 33 **(E)** 34

2013B

24. A positive integer n is *nice* if there is a positive integer m with exactly four positive divisors (including 1 and m) such that the sum of the four divisors is equal to n. How many numbers in the set $\{2010, 2011, 2012, \dots, 2019\}$ are nice?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

2002B

21. Andy's lawn has twice as much area as Beth's lawn and three times as much area as Carlos' lawn. Carlos' lawn mower cuts half as fast as Beth's mower and one third as fast as Andy's mower. If they all start to mow their lawns at the same time, who will finish first?

(A) Andy **(B)** Beth **(C)** Carlos **(D)** Andy and Carlos tie for first.

(E) All three tie.

2007A

23. How many ordered pairs (m, n) of positive integers, with $m > n$, have the property that their squares differ by 96?

(A) 3 **(B)** 4 **(C)** 6 **(D)** 9 **(E)** 12

2008A

22. Jacob uses the following procedure to write down a sequence of numbers. First he chooses the first term to be 6. To generate each succeeding term, he flips a fair coin. If it comes up heads, he doubles the previous term and subtracts 1. If it comes up tails, he takes half of the previous term and subtracts 1. What is the probability that the fourth term in Jacob's sequence is an integer?

(A) $\frac{1}{6}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{5}{8}$ **(E)** $\frac{3}{4}$

2012A

22. The sum of the first m positive odd integers is 212 more than the sum of the first n positive even integers. What is the sum of all possible values of n?

(A) 255 **(B)** 256 **(C)** 257 **(D)** 258 **(E)** 259

2003B

24. The first four terms in an arithmetic sequence are $x+y$, $x-y$, xy, and x/y, in that order. What is the fifth term?

(A) $-\frac{15}{8}$ **(B)** $-\frac{6}{5}$ **(C)** 0 **(D)** $\frac{27}{20}$ **(E)** $\frac{123}{40}$

2004A

24. Let $a_1, a_2, \cdots$, be a sequence with the following properties.

(i) $a_1 = 1$, and

(ii) $a_{2n} = n \cdot a_n$ for any positive integer n.

What is the value of $a_{2^{100}}$?

(A) 1 **(B)** 2^{99} **(C)** 2^{100} **(D)** 2^{4950} **(E)** 2^{9999}

2015B

24. Aaron the ant walks on the coordinate plane according to the following rules. He starts at the origin $p_0 = (0,0)$ facing to the east and walks one unit, arriving at $p_1 = (1,0)$. For $n = 1, 2, 3, \ldots$, right after arriving at the point p_n, if Aaron can turn 90° left and walk one unit to an unvisited point p_{n+1}, he does that. Otherwise, he walks one unit straight ahead to reach p_{n+1}. Thus the sequence of points continues $p_2 = (1,1)$, $p_3 = (0,1)$, $p_4 = (-1,1)$, $p_5 = (-1,0)$, and so on in a counterclockwise spiral pattern. What is p_{2015}?

(A) $(-22,-13)$ **(B)** $(-13,-22)$ **(C)** $(-13,22)$ **(D)** $(13,-22)$

(E) $(22,-13)$

2016B

24. How many four-digit positive integers $abcd$, with $a \neq 0$, have the property that the three two-digit integers $ab < bc < cd$ form an increasing arithmetic sequence? One such number is 4692, where $a = 4$, $b = 6$, $c = 9$, and $d = 2$.

(A) 9 **(B)** 15 **(C)** 16 **(D)** 17 **(E)** 20

2007A

25. For each positive integer n, let $S(n)$ denote the sum of the digits of n. For how many values of n is $n + S(n) + S(S(n)) = 2007$?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

2010A

25. Jim starts with a positive integer n and creates a sequence of numbers. Each successive number is obtained by subtracting the largest possible integer square less than or equal to the current number until zero is reached. For example, if Jim starts with $n = 55$, then his sequence contains 5 numbers:

$$\begin{aligned} &55 \\ 55 - 7^2 &= 6 \\ 6 - 2^2 &= 2 \\ 2 - 1^2 &= 1 \\ 1 - 1^2 &= 0 \end{aligned}$$

Let N be the smallest number for which Jim's sequence has 8 numbers. What is the units digit of N?

(A) 1 **(B)** 3 **(C)** 5 **(D)** 7 **(E)** 9

2011B

25. Let T_1 be a triangle with sides 2011, 2012, and 2013. For $n \geq 1$, if $T_n = \triangle ABC$ and D, E, and F are the points of tangency of the incircle of $\triangle ABC$ to the sides AB, BC, and AC, respectively, then T_{n+1} is a triangle with side lengths AD, BE, and CF, if it exists. What is the perimeter of the last triangle in the sequence (T_n)?

(A) $\frac{1509}{8}$ **(B)** $\frac{1509}{32}$ **(C)** $\frac{1509}{64}$ **(D)** $\frac{1509}{128}$ **(E)** $\frac{1509}{256}$